ICP Variants Robustness to Gaussian and Impulsive Noise

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Abstract

Iterative closest point (ICP) is a 3D surface-based rigid registration algorithm that locates and uses corresponding point pairs to compute the registration transformation. In the registration of 3D surfaces ICP assumes outlier free data, using all the corresponding point pairs regardless of their distance from the data surface to the closest point in the model surface. Computed tomography (CT) data was acquired from four anatomical phantoms. Two outlier robust variants of ICP, Haralick ICP and Trimmed ICP, were investigated to evaluate their robustness to Gaussian and impulsive noise. The results show that Trimmed ICP produces an accurate registration for both noise types but is computationally expensive.

1 Introduction

Diagnosis and treatment planning of specific diseases requires image acquisition from a range of 3D volumetric modalities including CT and magnetic resonance (MR) [1]. Multimodal data provides structural and functional information adding extra value to each of the modalities involved. Combining image data requires image registration to compute data alignment. ICP is a surface-based rigid registration algorithm that assumes outlier free data, reducing its robustness with real world data [2]. Haralick ICP (HICP) is an outlier robust method that assigns a weight to every corresponding point pair for pose estimation [3]. Corresponding point pairs with the largest residual error are assigned a minimal weighting value to limit impact on pose estimation. Trimmed ICP (TrICP) sorts the distances between each point in the data surface and the closest point in the model surface, selecting a subset of the closest point pairs to compute the registration [4].

In this investigation two CT surfaces, from four anatomical phantoms are registered, with the data surface having different levels of Gaussian and impulsive noise added to simulate outliers. The emphasis of this investigation is to establish the effect that outliers, simulated by Gaussian and impulsive noise, has on the registration of ICP variants.

2 Overview of ICP Variants

2.1 ICP

ICP is a representation independent algorithm that uses point sets to compute rigid registration transformations between two surfaces. The largest volume or highest resolution surface is assigned as the "model" surface (static during registration) with the remaining surface assigned as the "data".

1. Assume that both surfaces are in point sets with the data surface, $\boldsymbol{P}$, having N_p points, $\{p_i,\ i = 1,\ldots,N_p\}$ and the model surface, $\boldsymbol{M}$, having N_m points, $\{m_j,\ i = 1,\ldots,N_m\}$. For each data point p_i find the closest point in $\mathbf{M}$:
$$d(p_i, M) = \min_{m \epsilon M} \|m - p_i\| \tag{1}$$
2. Using the correspondence pairs, compute the optimal transformation, rotation ($\boldsymbol{R}$) and translation ($\boldsymbol{T}$) to minimise the point pairs' mean square error (MSE), where $\widehat{m}_i$ is the point in $\boldsymbol{M}$ closest to p_i:
$$MSE = \frac{1}{N_p} \sum_{i=1}^{N_p} \|\widehat{m}_i - \boldsymbol{R}(p_i) - \boldsymbol{T}\|^2 \tag{2}$$

Optimal transformation estimation uses closed-form solutions such as quaternion and singular value decomposition (SVD), providing transformation at iteration k with rotation and transformation ($\boldsymbol{R}_k$ and $\boldsymbol{T}_k$). Apply the transformation matrix to data, $\boldsymbol{P}_0$:
$$P = \boldsymbol{R}_k * P_0 + \boldsymbol{T}_k \tag{3}$$

3. If the change in MSE is less than a predefined threshold or the maximum number of iterations has been reached, terminate the algorithm. Otherwise start the next iteration and continue until one of the terminating conditions is reached.

ICP assumes outlier free data, which is not possible in real world applications and has no preventive measures in dealing with outliers making it sensitive to outliers.

2.2 HICP

HICP [3] considers the outlier problem by assigning weights w_i to point pairs, minimising:
$$\Sigma^2 = \sum_{i=1}^{N} w_i \|\widehat{m}_i - (\boldsymbol{R}p_i + \boldsymbol{T})\|^2 \tag{4}$$

Weight values are calculated using the iterative weighted least-squares pose estimation method with outliers having a zero weighting and no influence on the registration:
$$w_i = \begin{cases} \left[1 - \frac{\|\varepsilon_i\|^2}{(cS)^2}\right]^2 & if\ \|\varepsilon_i\| \leq cS \\ 0 & otherwise \end{cases} \tag{5}$$

where $\varepsilon_i = \widehat{m}_i - (\boldsymbol{R}p_i + \boldsymbol{T})$ is the residual error; c is a tuning constant, and S is a scale estimator equivalent to the median of absolute distance of the entire point clouds [5].

2.3 TrICP

TrICP [4] assumes that that a minimum number of the points in the data surface can be successfully paired with a model point (minimum overlap). Any other point pairs beyond the minimum overlap are deemed as outliers and are excluded from pose estimation. TrICP computes the squared distance for each corresponding pair sorting them in ascending order. The minimum overlap value, ξ, is found by minimising $\psi(\xi)$ in the interval [0.4, 1.0] using a Golden Selection Search where the number of paired data points $N_{po} = \xi N_p$

$$\psi(\xi) = \frac{\sum_{i=1}^{N_{po}} d_{i:Np}^2}{N_p * \xi * \xi^{1+\lambda}} \quad (6)$$

The N_{po} points are used to compute the sum of the least trimmed squares distance (S'_{LTS}) of the closest N_{po} points between the data and model surface. An optimal transformation is computed using the N_{po} points minimising S'_{LTS}.

The purpose of this investigation is to establish whether ICP, TrICP and HICP are resilient to Gaussian and impulsive noise and the impact that the noise will have on the overall registration accuracy, number of iterations and the time for convergence.

3 Methodology

3.1 Image Acquisition

Four anatomical phantoms were selected for CT data acquisition, a polystyrene mannequin head; a plastic head containing tissue equivalent and real bone; a Perspex moulded foot containing real bone; and a silicon foot with 3 simulated surface ulcers (Figure 1). CT data was acquired using a Philips Brilliance 10 Slice System with a slice thickness of 1mm (www.philipsmedical.com). The scanner bed was removed from the data using the marching cubes algorithm in Analyze 10.0 (Mayo Clinic, Rochester) and 3D surfaces were extracted and stored in the stereolithography (STL) format.

(a) (b) (c) (d)

Figure 1: The anatomical phantoms used in data acquisition (a) polystyrene head model; (b) plastic head model; (c) Perspex moulded foot; (d) silicon foot with 3 simulated ulcers.

3.2 Experimental Design

The robustness of ICP, TrICP and HICP to Gaussian and impulsive noise was investigated. For each phantom, a new set of surfaces were created with added Gaussian and impulsive noise and rotated 5° in each orthogonal axes. All surfaces were decimated to 10% of their original facets and the noisy surfaces were registered with the original surfaces using ICP, TrICP and HICP algorithms implemented in Matlab 2009b. The addition of the Gaussian and impulsive noise is similar to the approach taken in [6].

Gaussian noise was added to the data surface with a signal-to-noise ratio of 10, 20, 30, 40 and 50 dB. Impulsive noise was added randomly to 10% and 20% of the data surfaces points. Furthermore different levels of impulsive noise were added to the data surface with the values of β tested being 0.1, 0.2, 0.3, 0.4 and 0.5 respectively.

In both investigations the termination criteria for the ICP variants were:

1. The maximum number of iterations, 300, was exceeded. This value was selected to give the algorithms sufficient iterations for convergence.

2. The change in error measure between iterations was less than $1x10^{-5}$mm.

The transformation matrix, number of iterations and convergence time were recorded. To measure the overall accuracy, the RMS between the registered non decimated noiseless surfaces was found. The RMS is the square root of the mean square distance between the closest points in the noiseless surface for every point in the noisy surface. The average convergence time, iteration and RMS were calculated.

4 Results and Discussion

4.1 Robustness to Gaussian Noise

Figure 2 (a), (b) and (c) show the impact of Gaussian noise on average RMS, time and iterations for a range of Gaussian noise levels. Results are averaged over the four phantoms with each individual phantom producing similar shape graphs, displaying similar performance. As SNR_dB decreases the level of noise increases. TrICP was the most accurate method, with RMS<0.1mm for all Gaussian levels, Figure 2(a). ICP and HICP produced similar RMS results except at 10dB where ICP had an RMS of 1.95mm and HICP had 1.76mm. From 30dB onwards all variants produced similar results <0.03mm but TrICP had a better RMS. TrICP required most iterations, even at low noise levels where the RMS with ICP and HICP is comparable, Figure 2(c). Furthermore TrICP is the slowest method except at 10dB (ICP slowest), Figure 2(b). Overall TrICP is considerably slower with HICP the fastest method for all levels of noise.

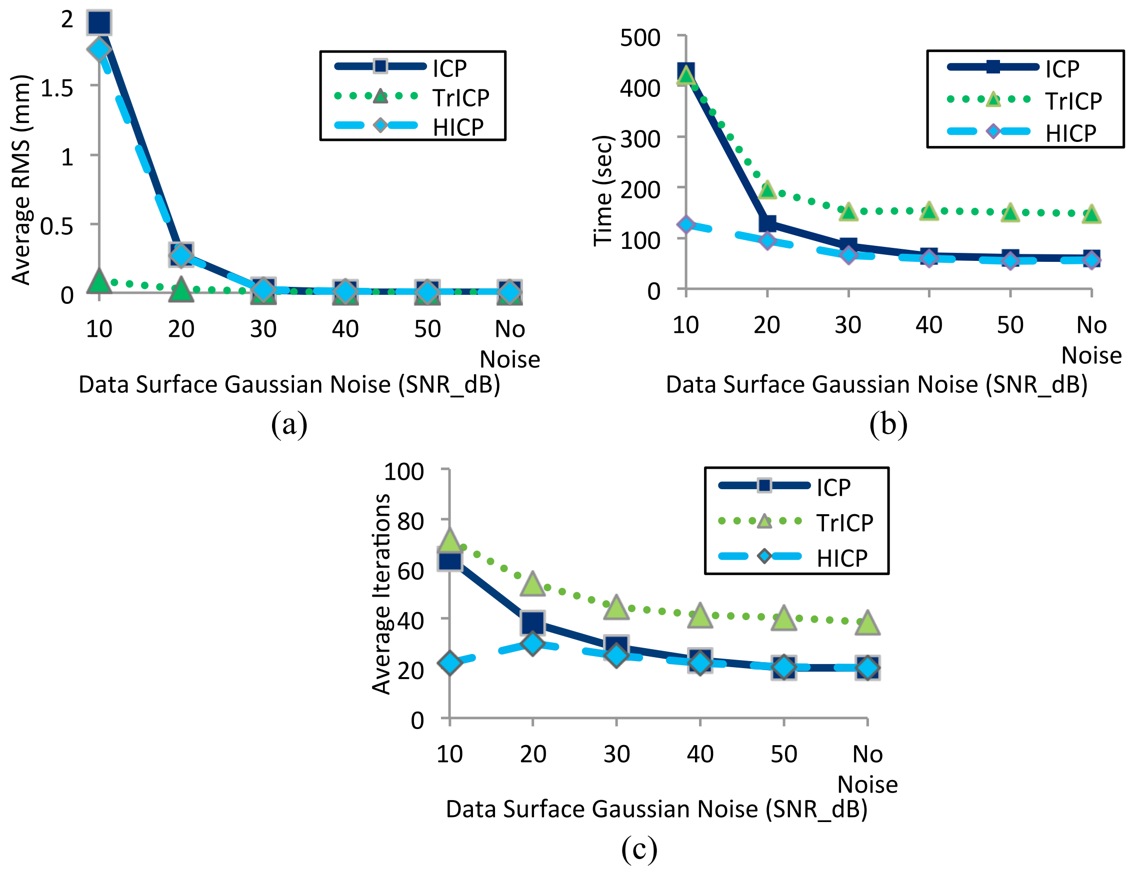

Figure 2. The effect of Gaussian noise on the average: (a) registration accuracy; (b) convergence time (sec); (c) iterations.

Although TrICP is effective in detecting outliers, this produces a high number of iterations and registration time, especially at low noise levels, showing that outlier detection is computationally expensive. Since RMS difference at low noise is negligible, consideration is required in using TrICP over HICP due to its computation requirements.

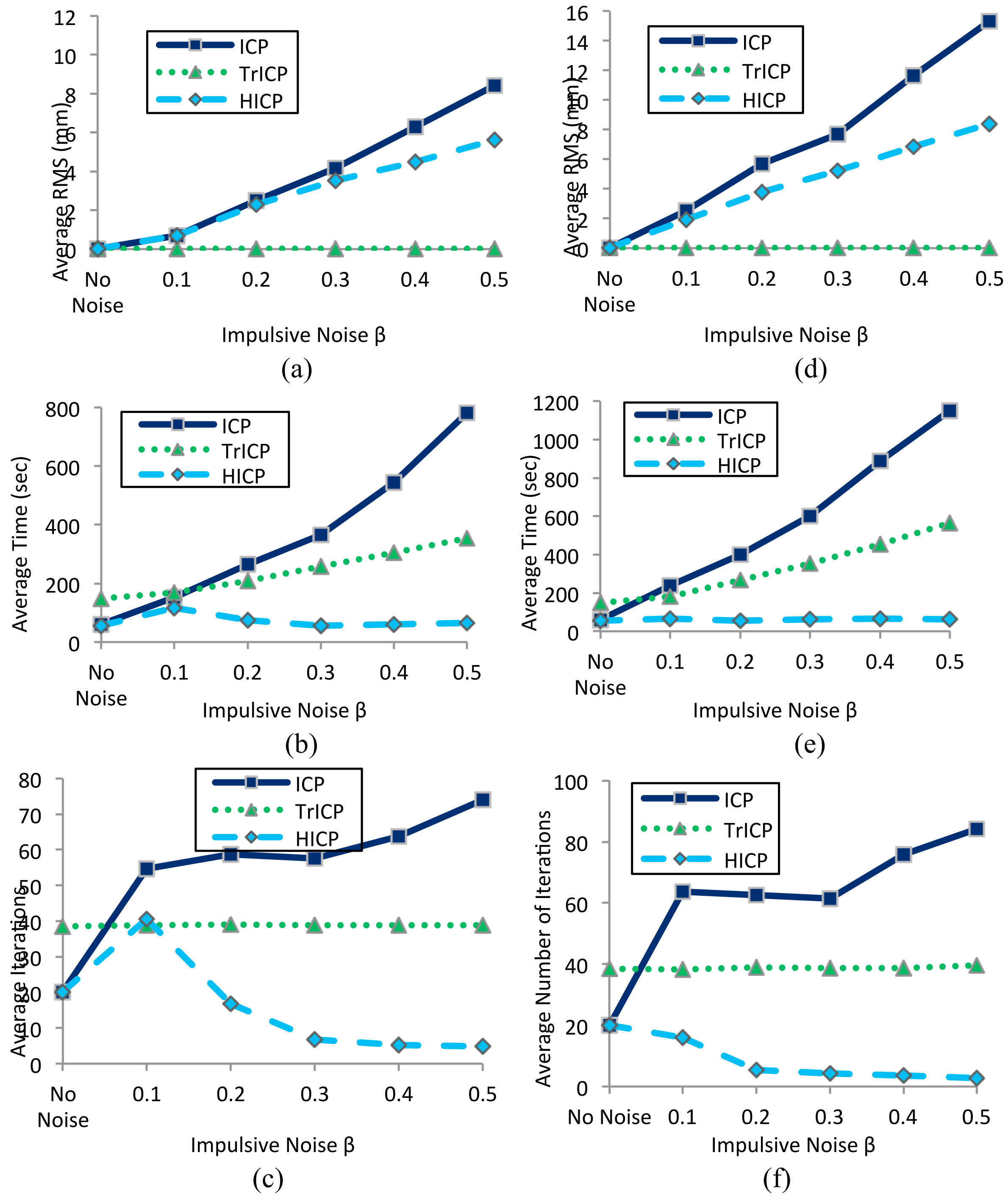

Figure 3. The effect of impulsive noise added to 10% of the data surface points on the average: (a) registration accuracy; (b) convergence time (sec); (c) iterations; and impulsive noise added to 20% of the data surface points on the average: (d) registration accuracy; (e) convergence time (sec); (f) iterations.

4.2 Robustness to Impulsive Noise

In the presence of impulsive noise in each phantom, regardless of the percentage of points changed or the noise level, TrICP produces the same RMS value showing its resilience to noise, Figure 3(a) and (d). ICP produces the largest RMS for surfaces with 10% (8.41mm) and 20% (15.32mm) of their points as outliers, showing no surface convergence. HICP performed slightly better than ICP but both methods are unsuitable for accurate registration (RMS >2mm) for impulsive noise levels of $\beta > 0.2$. TrICP requires a similar number of iterations regardless of noise, whilst HICP generally requires the fewest, decreasing as the noise level increases, indicating that HICP may get stuck in local minima with high levels of noise, Figure 3(c) and (f). ICP requires the most iterations once noise is added to the surface showing the significant impact that the noise has on the registration and the high RMS returned. HICP is the fastest of the methods, due to its small number of iterations, with ICP taking the most time when noise is added to the surface, Figure 3(b) and (e).

With impulsive noise addition TrICP is the method of choice since the RMS level does not change even with high noise and a higher percentage of points changed. Although HICP is the fastest method it is not effective in producing a suitable registration for medium to high levels of impulsive noise. At the highest impulsive noise level with 20% outliers the average HICP convergence time is 61 seconds compared with TrICP, 564 seconds, and ICP with 1152 seconds, Figure 3(e).

5 Conclusions

This investigation has shown that TrICP is robust in rejecting outliers from Gaussian and impulsive noise, but is computationally expensive so careful consideration must be given to the trade-off between required registration accuracy and the time available for registration. HICP is a viable alternative in terms of time but is only acceptable for low noise levels. ICP cannot reject outliers and is unsuccessful in registering noisy data. TrICP could be used with a manual transformation to bring both surfaces close to alignment prior to registration without the need for pre-registration data cleansing. This would remove the requirement for a landmark based registration and the issues associated with it.

References

[1] J. K. Iglehart, “The new era of medical imaging progress and pitfalls,” New England Journal of Medicine, vol. 354, no. 26, pp. 2822-2828, 2006.

[2] J. M. Phillips, R. Liu, and C. Tomasi, “Outlier Robust ICP for Minimizing Fractional RMSD,” 2006.

[3] Z. Mao, P. Sebert, and A. Ayoub, “Development of 3D measuring techniques for the analysis of facial soft tissue change,” in Medical Image Computing and Computer-Assisted Intervention–MICCAI 2000, 2000, pp. 1051-1060.

[4] D. Chetverikov, D. Svirko, D. Stepanov, and P. Krsek, “The trimmed iterative closest point algorithm,” in International Conference on Pattern Recognition, 2002, vol. 16, pp. 545-548.

[5] R. Haralick, H. Joo, C.-N. Lee, X. Zhuang, V. G. Vaidya, and Mn. B. Kim, “Pose Estimation from Corresponding Point Data,” IEEE Transactions on Systems, Man and Cybernetics, vol. 19, no. 6, pp. 1426-1446, 1989.

[6] A. Almhdie, C. Leger, M. Deriche, and R. Ledee, “3D registration using a new implementation of the ICP algorithm based on a comprehensive lookup matrix: Application to medical imaging,” Pattern Recognition Letters, vol. 28, no. 12, pp. 1523-1533, Sep. 2007.