Non-invasive Cardiac Electro-physiology Imaging: Boundary Element Formulation for the Forward and Inverse Model

Yuan Ruan [a*]Simon Arridge [a], Maxime Sermesant [b],
Phani Chinchapatnam [a], Kawal Rhode [b] and Reza Razavi [b]

[a]Center for Medical Image Computing, Computer Science Department, University College London
[b] Cardiovascular Modelling in the Division of Imaging Sciences, Guy and St Thomas' Hospitals,
King's College London, Guy and St Thomas' Hospitals

Abstract. Visualization of the electrical activities of the heart can be considered as an imaging modality, which merges the electrical data (e.g. ECG) and the anatomical image data (MR/CT) thus giving a better manifestation of heart behaviour for clinical applications. Non-invasive imaging of the electrical activity of heart visualises the heart surface potential distributions spatially and temporally from the measured body surface potentials. In this paper, we describe the numerical techniques involved in building a 4-sphere forward model with homogeneous regions representing different thorax compartments. Patient-specific anatomy geometry can be get from MR/CT volumes using image segmentation techniques (not in the scope of this paper) and discretizing into volumetric or surface meshes depending on different methods choose to calculating electrical potentials. Regularisation techniques used in inverse computation of epicardial electrical potentials is zero-order Tikhonov regularisation with automatic and efficient parameter selection method using L-curve. Results from two spherical cases and realistic heart-torso geometry validations have been satisfactory both in terms of correlation and iso-potential plots.

1 Introduction

Visualization of the electrical activities of the heart can be considered as an imaging modality. It merges the electrical data (e.g. ECG) and the anatomical image data (MR/CT) thus giving a better manifestation of heart behaviour for clinical applications. Non-invasive imaging of the electrical activity of the heart requires to solve and visualise the heart surface potential distributions spatially and temporally from the measured body surface potentials. In other words, to solve the inverse electrocardiography problem. Techniques relating the electrical potential distributions among the heart surface and the body surface include Catheter technique, Angioplasty Balloon Catheter Inflation technique, electrocardiography (ECG) and Body Surface Potentials Mapping technique. They have been studied and improved in clinical applications to assist with both diagnosis and intervention of pathological heart conditions. Todays pre-operative planning of ablation for treating arrhythmia is still invasive, thus non-invasive techniques which can accurately relate heart and body surface electrical activities are preferable in clinical applications and are likely to replace the invasive measuring system in the future.

In the literature of cardiac electro-physiology, there are different kinds of volume conductor models relating body surface potentials to heart surface potentials, such as concentric spheres model [1,2]; homogeneous torso models [3] which considers only one conductivity within torso; and the inhomogeneous torso models [4–6], which considers the conductivities of all tissue compartments; and the anisotropic torso models [7,8] which considers the anisotropic conductivity of heart and/or skeletal muscle. There are Potential-based imaging and Activation time imaging. Potential-based imaging refers mainly to Body Surface Potential Mapping (BSPM), which reconstructs epicardial potentials spatially. While Activation time imaging estimates local activation time over the whole epicardium surface from BSPM results, it is receiving increasingly attention in recent years. The bi-domain model [9] or a uniform dipole layer [10] have been used to compute activation time. G. Fischer and M.Seger [7] have made studies on inhomogeneities as well. Combining both the potential-based and activation time imaging method, Electrocardiography Imaging(ECGI) technique [11] has been proposed, in which the potential map, activation map (isochrones) and electrogram is computed and visualized at the same time. Other similar studies [12] as well as ECGI eventually all adopted a homogeneous torso model and obtain human heart-torso geometry from other imaging modalities though there are still controversies of whether or not should the anisotropic conductivities and should a multiple torso compartments be taken into consideration.

In this paper, we describe the numerical techniques involved in building a simple 4-sphere forward model with homogeneous regions representing different thorax compartments. Regularisation techniques used in inverse computing epicardial electrical potentials is zero-order Tikhonov regularisation with automatic and efficient parameter selection method using L-curve. Results from 2 cases of spherical validations have been satisfactory while validations have also been done on the realistic geometry.

*Center for Medical Image Computing, Computer Science Department, University College London, WC1E 6BT

2 Methods

Human thorax can be treated as volume conductor, for the problem of bioelectrical volume conduction, we need to solve the Laplace's Equation, for its solution characterize the behaviour of electrical potentials inside biological environment. Assuming tissues are isotropic, for each compartment (lungs, heart) we have a Laplace's equation to govern the potential behaviours according to the theory of the Quasi-static Maxwell's equations due to low-frequency response of human tissue.

$$\nabla \cdot (\sigma_k \nabla U_i) = \sigma_k \nabla^2 U_i = 0 \tag{1}$$

where σ_k is the conductivities for each compartments when k = t, L, R, e respectively representing torso, left, right lung and epicardium; i = 1, 2, 3, 4; with boundary conditions as follow, we define the potentials on each conductivity interface:

$$\begin{array}{lll} U_1 & := & U_t \mid_{\Gamma_1} \\ U_2 & := & U_t \mid_{\Gamma_2} = U_L \mid_{\Gamma_2} \\ U_3 & := & U_t \mid_{\Gamma_3} = U_e \mid_{\Gamma_3} \\ U_4 & := & U_t \mid_{\Gamma_4} = U_R \mid_{\Gamma_4} \end{array} \tag{2}$$

Because the current should be continuous across each conductivity interface, Neumann Conditions are defined as:

$$\begin{array}{lllll} J_1 & := & \sigma_t \frac{\partial U_t}{\partial n} \mid_{\Gamma_1} & = 0 & \\ J_2 & := & \sigma_L \frac{\partial U_L}{\partial n} \mid_{\Gamma_2} & = -\sigma_t \frac{\partial U_t}{\partial n} & \mid_{\Gamma_2} \\ J_3 & := & \sigma_e \frac{\partial U_e}{\partial n} \mid_{\Gamma_3} & = -\sigma_t \frac{\partial U_t}{\partial n} & \mid_{\Gamma_3} \\ J_4 & := & \sigma_R \frac{\partial U_R}{\partial n} \mid_{\Gamma_4} & = -\sigma_t \frac{\partial U_t}{\partial n} & \mid_{\Gamma_4} \end{array} \tag{3}$$

where n is the outward normal vector of each boundary. Making use of Green's theorem (also known as the Divergence Theorem), the boundary Integral equation that we would like to solve is:

$$\begin{array}{l} c(m)u(m) - \int_\Gamma (u \dfrac{\partial G}{\partial n} - G \dfrac{\partial u}{\partial n}) d\Gamma = 0 \\ where\, c(m) = \begin{cases} 1\, m \in \Omega \\ \frac{1}{2}\, m \in \Gamma \end{cases} \end{array} \tag{4}$$

For each of the surfaces involved we have a corresponding integral equation, therefore a set of boundary integral equations is obtained and can be written in an assembly form:

$$A \cdot U = S \tag{5}$$

A is a general transfer matrix, potentials vector U includes potentials on all surfaces if there are multiple regions with different conductivities considered for the thorax volume conductor, and S denotes the equivalent source.

To solve the potential from the surface integral equations, Boundary element discretization techniques have been used [13] which includes the surface tessellation and use of a linear combination of basis functions to approximate the boudary potentials.

2.1 Forward Model

A forward model has been developed based on Helsinki BEM library [14]. It consisted of 4 spheres respectively modelling two lungs, the heart and the torso is shown in Figure 1. Different conductivity values were assigned to the 4 regions and this model assumes a dipole source on the origin to mimic a realistic electrical potential effect from heart on the body surface.

2.2 Inverse Model

The inverse problem of cardiac electro-physiology is defined as to determine the epicardial surface potential distribution from the measured body surface potentials. Due to the illposed-ness property of inverse problems, a very small variation on the input (noises) will largely alter the output results (epicardial potentials). Therefore, regularisation techniques are required during the inverse computing. It is also very important to choose the regularisation parameter. The L-curve method [15] is adopted because it is a good balance of the solution and residual error. Start from Equation 5,the measurement is made on body surface U_2, and we want to solve for epicardial potential U_1:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} S \\ 0 \end{bmatrix}, \tag{6}$$

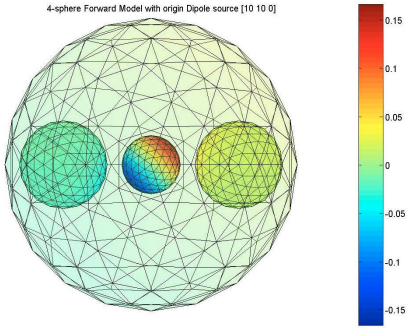

Figure 1. 4-sphere forward model: the middle sphere represents epicardial surface; two other inner spheres represents two lung surfaces; the outer sphere represents torso surface, the middle and the outer spheres are concentric

We solve the linear system by eliminating S, the relationship between U_1 and U_2 can be obtained as

$$U_1 = (A_{11} - A_{12}A_{22}^{-1}A_{21})^{-1}(A_{12} - A_{11}A_{21}^{-1}A_{22})U_2, \tag{7}$$

or more generally when matrix A_{21} is not a squared matrix:

$$U_1 = (A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1}A_{21}(I - A_{11}^{-1}A_{12}A_{22}^{-1}A_{21})U_2, \tag{8}$$

which is used when the number of body surface nodes is different from that of heart surface.

Let M denotes the measurement operator, by multiplying with body surface potentials, picks some random measurement points on the body surface mimicking electrodes measurements, and this serves as the input to the forward model. The data we use as the input to the forward model denotes as $Y = M \cdot U_2$, and a possible formulation of the inverse computing of the epicardial potential problem under the ill-posed-ness nature of inverse problem is to use pseudo inverse, however it could not provide us a satisfactory results under the above inverse formulation. Therefore, a zero-order Tikhonov regularisation has been used, in which by adding a penalty term λR, where R is the identity matrix in zero-order Tikhonov regularisation. The formulation of the zero-order Tikhonov regularisation for solving the following minimisation problem:

$$\arg\min_{U_1 \in R} |Y - MHU_1|^2 + \lambda I, \tag{9}$$

where $H = (A_{12} - A_{11}A_{21}^{-1}A_{22})^{-1}(A_{11} - A_{12}A_{22}^{-1}A_{21})$ and $Y = M \cdot U_2$,

is:

$$U_1 = (H^T H + \lambda I)H^T Y. \tag{10}$$

3 Results and Discussion

3.1 Validation on spheres

Based on building the 4-sphere forward model with single dipole source at the origin, two circumstances of the inverse calculation have been validated. One is with the limited number of measurement points on body surface (take randomly half of the points shown in Figure 3) which mimics the limited number of body surface electrode measurements; the other is adding white noises further to the limited measurements. The recovered epicardial potential were unsatisfied when no regularisation procedure was used, but the after adopting the zero-order Tikhonov regularisation technique, the result is acceptable.

L-curve is one of the methods for selecting appropriate regularisation parameter, it selects the corner point with a maximum curvature on the log-log plot of residual squared norm and the solution squared norm. In order to make an automatic selection of the regularisation parameter, we first select in a coarse series of parameters, then create an spline interpolation among neighbour points followed by once again localising the maximum curvature point so as to get an accurate parameter (shown in Figure 3 (b)). The calculated results of two scenarios were plotted with point

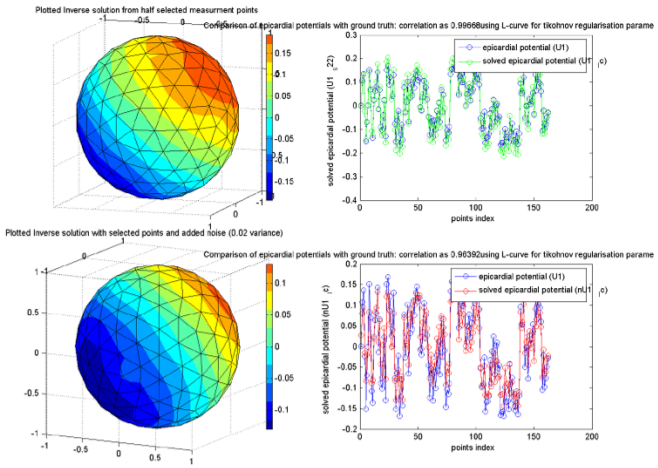

Figure 2. plotted results of recovered potentials on heart sphere, Case 1 (upper): limited number of measurements, Case 2 (lower): further added noise.

number comparison with ground truth and plotted on the surfaces as shown in Figure 2. Cross correlations between the recovered potentials and the corresponding ground truth were calculated respectively as 0.99668 and 0.96392 which are sufficiently satisfactory before moving to next level of validations on the realistic data.

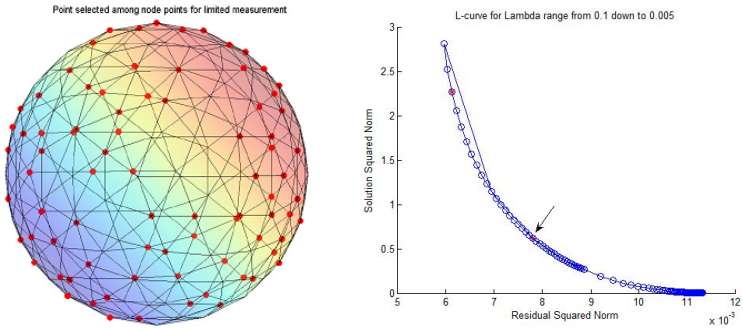

Figure 3. (a)Left: Selected limited measurement points on body surface to mimic the realistic discrete measurements; (b)Right: L-curve for the regularisation parameters ranged from 0.005 to 0.1

3.2 Validation on realistic geometry

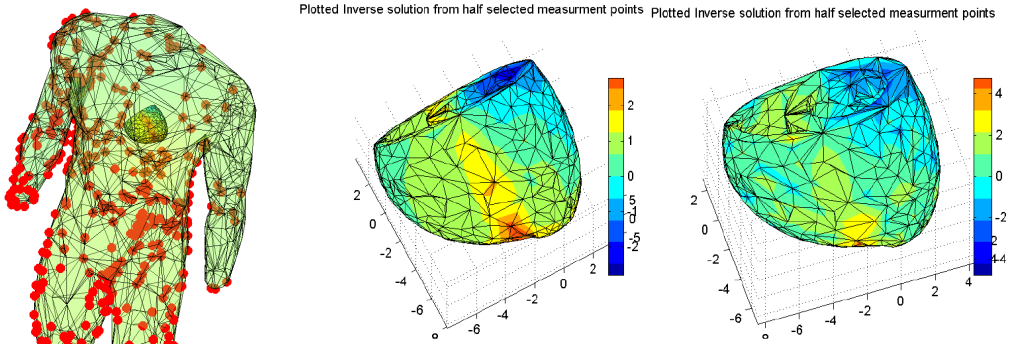

Figure 4. (a) Left: human realistic heart-torso geometry with potentials calculated from forward model and selected measurement points; (b) Right: recovery results compared with forward model ground truth

Validation has been made on realistic data with human anatomy geometry [16]as shown in Figure 4. Similarly as what has been done in the spherical validation, half number of the points on the torso surface were selected, and the recovery result was compared to the ground truth constructed from the forward model as shown in Figure 4 (b). The plotted iso-potentials shows that the recovered the minima point at the top left corner and the maxima point at the bottom of the epicardial surface, which were similar compared to the ground truth. The cross correlation between the recovery

results and the ground truth is 0.85130, which is not as good as that (0.99668) in the sphere model, which may due to the creation of the ground truth potentials on the heart surface are calculated from potential values on nodes from the volumetric heart mesh. This mapping from volumetric mesh (the original data)to surface mesh may contributes to the error effects, made it not as good as what come out of spherical models, but still the results on realistic data are acceptable.Therefore, we can conclude that the results from two cases of spherical validations have been satisfactory while results on a realistic geometry are of less satisfactory in terms of correlation value but visually acceptable in terms of iso-potential plots. Possible improvements on the results can be achieved by building the forward model using finer heart-torso mesh, also by selecting body surface measurements not randomly but with certain patterns (i.e. equally spaced around chest), which requires further adjustments on the mesh generation, etc.

4 Future works

Further works have been planned on validations on clinical data due to be taken at one of our collaborative hospital. Clinical level of validation requires full-chest MR scan of the patient and body surface potential mapping system which provides the measured electrical potentials. During the regular electro-physiological study, catheter feedback epicardial potentials provide the coarse ground truth, which can be used in the validation of the calculated epicardial potentials measured from body surface potential mapping system. Furthermore, segmentation techniques are needed for constructing surface meshes from full-chest MR scan.

Acknowledgments

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