Robust Multiview Registration of 3D Surfaces via ℓ_1-norm Minimization

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Registration (or alignment) of surfaces is an important task in many areas such as robotics, computer aided modeling, virtual reality and surface reconstruction. Building a full model of a object requires registering multiple surfaces observed from different viewpoints. In this paper we present an approach robust to outliers, for registering multiple surfaces simultaneously when the transformation is rigid.

Multiview registration simultaneously registers multiple scans. Given scans $\mathcal{S}_1 \dots \mathcal{S}_N$ and a view-graph (i.e. a graph with edges between scans which have common points), $\mathcal{G} = (\{1 \dots N\}, E)$. This problem can be generalized as

$$\min_{\mathbf{M}_i \in SE(3)} \mathcal{E}(\mathbf{M}_1 \dots \mathbf{M}_N) \tag{1}$$

Where

$$\mathcal{E}(\mathbf{M}_1 \dots \mathbf{M}_N) = \sum_{(i,j)\in E} \sum_{\mathbf{x}\in\mathcal{S}_i} d(\mathbf{M}_i\mathbf{x}, \mathbf{M}_j\mathcal{S}_j)^2 \tag{2}$$

and $\mathbf{M}\mathcal{S} = \{\mathbf{M}\mathbf{x} \,|\, \mathbf{x} \in \mathcal{S}\}$.

The iterative closest point algorithm [1, 3] (ICP) is a pairwise alignment method which iteratively solves the optimization problem (2) by splitting the problem into a correspondence estimation step and a transformation estimation step.

Correspondence Estimation: The correspondence estimation problem is to find a point in $\mathbf{y}_i \in \mathcal{S}_2$ for each $\mathbf{x}_i \in \mathcal{S}_1$. When the scans are close enough the correspondences can be calculated using closest point queries [1] as above. If the scans are far apart correspondences can be found using feature matching. In either case the matches are not always accurate. For instance when the scans are partially overlapping, some points in the scans will not have matching points. Hence, heuristics are applied for better alignment *e.g.* removing correspondences which are too far away; see Rusinkiewicz and Levoy [6] for more details.

Transformation Estimation: Once the correspondences are known, the transformation can be estimated by minimising a sum of squared distance function

$$\{\mathbf{M}_i^{(s)}\} = \underset{\{\mathbf{M}_i\}\subset SE(3)^N}{\arg\min} \sum_{(i,j)\in E} \sum_{k=1}^{n_{ij}} d(\mathbf{M}_i\mathbf{x}_{ij}^k, \mathbf{M}_j\mathbf{x}_{ji}^k)^2 \tag{3}$$

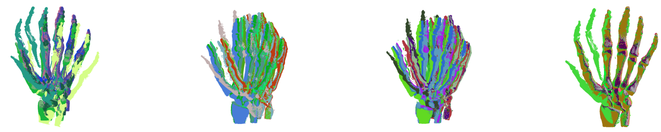
The distance function $(\mathbf{x},\mathbf{y})$ varies based on the approximation of $d(\mathbf{x},\mathcal{S})$ used. Commonly used are the euclidean distance (or point-to-point distance) and the distance between the point $\mathbf{x}$ and the tangent plane of $\mathcal{S}$ containing $\mathbf{y}$.

Recent works include Krishnan et al. [5], Govindu and Pooja [4] and Torsello et al. [7]. Krishnan et al. [5] solves (3), with d being the euclidean distance, using a manifold based Newton's method. While Govindu and Pooja [4] and Torsello et al. [7] find the ICP estimates for the pairwise transformation and obtain global transformations by global averaging. They use the consistency constraint $\mathbf{M}_{ij} = \mathbf{M}_j\mathbf{M}_i$ and find the global motions by solving the minimization problem for some distance d

$$\min \sum_{i,j} d(\mathbf{M}_{ij}, \mathbf{M}_j\mathbf{M}_i)^2 \tag{4}$$

These algorithms are dependent on finding matching points between scans, but a significant number of them are spurious, and it is necessary to clean up the matches obtained. This requires a substantial amount of tuning of parameters and the final result might still contain outliers. Since the number of outliers are sparse we formulate the registration optimization using the ℓ_1-norm. We present experimental results to show that the performance of our algorithm is comparable to state of the art algorithms.

If we assume that the residual noise in the scans are Gaussian we can model the transformation estimation step of multiview registration problem as



ICP initialization | Torsello et al. | ℓ_2-MVICP | ℓ_1-MVICP

Figure 1: The results here show the performance of different algorithms when the initialization (left) is bad. Torsello et al. [7] and ℓ_2-MVICP Krishnan et al. [5] both fail while ℓ_1 MVICP ignores the outlier scan and registers the rest of them correctly.

$$\mathbf{e}_{ij}^k = \mathbf{M}_i\mathbf{p}_{ij}^k - \mathbf{M}_j\mathbf{p}_{ji}^k \sim \mathcal{N}(0, \sigma^2) \tag{5}$$

Thus, the maximum likelihood estimate gives us ℓ_2 optimization problem which was solved by Krishnan et al. [5]. We instead formulate the problem in terms of outliers and assume that the noise is small (and not necessarily Gaussian)

$$\mathbf{M}_i\mathbf{p}_{ij}^k - \mathbf{M}_j\mathbf{p}_{ji}^k = \mathbf{O}_{ij}^k \tag{6}$$

Since the vector $\mathbf{O}_{ij} = [\mathbf{O}_{ij}^{k\top} \dots \mathbf{O}_{ij}^{n_{ij}\top}]^\top$ is sparse we can use the ℓ_1-norm to impose this constraint. So our cost function is

$$\mathcal{E}_{\ell_1}(\mathbf{M}_1 \dots \mathbf{M}_N) = \sum_{(i,j)\in E} ||\mathbf{O}_{ij}||_1 \tag{7}$$

By using the approximation $\mathbf{M}_i = \mathbf{I} + \mathbf{m}_i$ where $\mathbf{m}_i$ is of the form

$$\mathbf{m} = \begin{bmatrix} 0 & -\omega_z & \omega_y & u_x \\ \omega_z & 0 & -\omega_x & u_y \\ -\omega_y & \omega_z & 0 & u_z \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

we can reduce (7) to

$$\mathcal{E}_{\ell_1}(\mathbf{x}) = ||\boldsymbol{\delta} - \mathbf{D}\mathbf{x}||_1 \tag{9}$$

where, $\mathbf{x} = [\boldsymbol{\omega}_1 \cdots \boldsymbol{\omega}_N \; \mathbf{u}_1 \cdots \mathbf{u}_N]^T$ which can be solved iteratively (for an available implementation see [2]).

It must be noted that the approximation (8) is valid only if $\boldsymbol{\omega}_i$'s and $\mathbf{u}_i$'s are small. Hence, the optimization cannot start from an arbitrary point. For most cases, a good starting point can be obtained by an initial pairwise ICP step. The implementation details are described in the paper. We also show that when there are outliers, our method does significantly better than the state-of-the-art algorithms.

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