Image Denoising via Improved Sparse Coding

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Recently, many denoising algorithms [1, 2] work on image patches, so it is useful to formulate a patch-based image model

$$y_i = x_i + \eta_i, \tag{1}$$

where x_i is the original patch intensity written in a vectorized form, y_i is the vectorized noisy image patch and η_i is a vectorized noise patch. Our goal is to design a denoising algorithm which can remove the noise from y_i.

This paper presents a novel dictionary learning method for image denoising, which removes zero-mean independent identically distributed additive noise from a given image. Our main contribution is to find that a lower bound of dictionary is related with the level of noise, and design a novel dictionary learning method with the purpose of image denoising.

We first introduce a traditional sparse coding algorithm. Let $X \in R^{N\times L}$ be the input matrix (each column is an input vector), let $D \in R^{N\times M}$ be the basis matrix also called dictionary (each column is a coefficient vector), and let $S \in R^{M\times L}$ be the coefficient matrix (each column is a coefficient vector). The optimization model about traditional sparse coding is

$$\hat{D} = arg\min_{D,S} \|X - DS\|_2^2 + \lambda \|S\|_1$$
$$s.t. \quad \|d_i\|_2^2 \le b, \quad i = 1, \cdots, M. \tag{2}$$

In the model (2), L_1 norm is to guarantee sparsity, and L_2 norm limitation on the columns of D can remove the scaling ambiguity, d_i denotes the i-th column of D. This particular formulation has been extensively studied [3].

This paper considers the noise influence, thus we consider adding additive restraints for getting a suitable dictionary for denoising. The input vector x_i can be represented by $D\alpha_i$, *i.e.*, $x_i = D\alpha_i$. So (2) is rewritten by

$$y_i = D\alpha_i + \eta_i. \tag{3}$$

For simplicity, D is assumed by an invertible matrix, *i.e.*, $D \in R^{N\times N}$. This assumption doesn't affect the theory analysis. So we get the noise coefficient as

$$\hat{\alpha}_i = D^{-1}y_i = \alpha_i + D^{-1}\eta_i, \tag{4}$$

where $D^{-1}\eta_i$ is considered as the noise term, its covariance matrix C of noise is

$$C = Cov(D^{-1}\eta_i) = \mathrm{E}[D^{-1}\eta_i\eta_i^T(D^{-1})^T]$$
$$= D^{-1}\mathrm{E}[\eta_i\eta_i^T](D^{-1})^T, \tag{5}$$

where $\mathrm{E}[\cdot]$ denotes the expected value, T represents transposition. In this paper, we consider two different noises.

1) Additive noise. If the noise is constant and uncorrelated, $\mathrm{E}[\eta_i\eta_i^T]$ is a diagonal matrix, and can be represented by $\sigma^2 I$, *i.e.*

$$\mathrm{E}[\eta_i\eta_i^T] = \sigma^2 I, \tag{6}$$

where σ is the standard deviation of the noise, I denotes unit matrix. The equation (6) can be rewritten as

$$C_1 = \sigma^2 D^{-1}(D^{-1})^T. \tag{7}$$

2) Multiplicative noise. This covariance of this kind of noise is direct proportion to original image, *i.e.*,

$$\mathrm{E}[\eta_i\eta_i^T] = Kdiag[y_i], \tag{8}$$

where K is constant. Hence, we get

$$C_2 = KD^{-1}\, diag[y_i](D^{-1})^T. \tag{9}$$

If we assume that the two types of noise are statistically irrelevant, $\mathrm{E}[\eta_i\eta_i^T]$ can be divided into two parts

$$C = C_1 + C_2 = \sigma^2 D^{-1}(D^{-1})^T + KD^{-1}diag[y_i](D^{-1})^T. \tag{10}$$

Through strictly mathematical derivation, we find that a lower bound of dictionary is related with the level of noise:

$$\|d_i\|_2^2 \ge a, \quad i = 1, \cdots, M, \tag{11}$$

where a is a constant that is dictated by σ. So the optimization model called improved sparse coding is proposed as

$$\hat{D} = argmin_D \|X - DS\|_2^2 + \lambda\|S\|_1,$$
$$s.t. \quad a \le \|d_i\|_2^2 \le b, \quad i = 1, \cdots, M. \tag{12}$$

This paper utilizes the denoising framework of [4], which proposes denoising algorithm by introducing how sparsity and redundancy are brought to use. We will describe the general idea of the proposed algorithm with an illustration shown in Figure 1. Experimental results suggest the proposed algorithm outperforms traditional sparse coding algorithm.

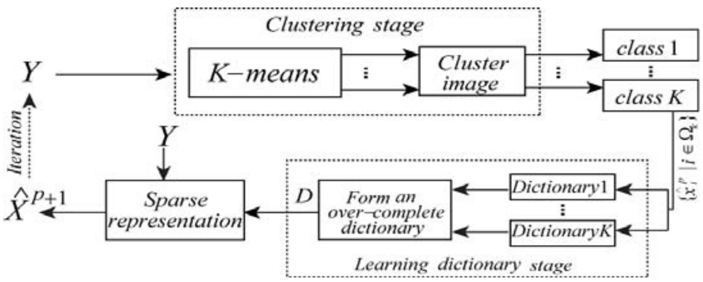

Figure 1: Illustration of sparse representation for image denoising.

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