Enhancing Gradient Sparsity for Parametrized Motion Estimation

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Most existing optic flow estimation approaches start from the constant intensity assumption which deduces an under-determined linear system. To solve this ill-posed problem, following the Horn-Schunck global model [3], many concepts and methods were proposed [4, 6].

Recently, inspired by the development in compressive sensing [2], Shen and Wu proposed a sparse model for optic flow estimation [5]. In this model, the wavelet transformation or the gradient is employed to expose the sparsity of the optic flow. As compressive sensing provides a systematical theory for solving ill-posed problems, the sparse model lays a foundation for estimating optic flow with efficient convex optimization.

In this paper, we propose a motion estimation framework according to the enhanced sparsity associated with gradients of the parametrized motion field. As Shen and Wu's model [5] is not sparse enough for motion with rotation and scaling, we suggest to enhance the sparsity of motion gradients by increasing the degree of freedom (DOF) of the parametrized motion model. With such an enhancement, we formulate the motion estimation as an ℓ_0 optimization problem. Along with an ℓ_1 norm regularization to the instant constancy assumption, this problem is solved by a reweighted ℓ_1 optimization approach. This framework provides a general and robust sparse model for optic flow estimation.

Let u and v be the horizontal and vertical motion components, respectively. Following Nir *et al.*'s notation [4], we employ the following parametrized motion model

$$\begin{aligned} u(x,y) &= \sum_{k=1}^{K} p_k(x,y)\varphi_k(x,y) \\ v(x,y) &= \sum_{k=1}^{K} p_k(x,y)\eta_k(x,y) \end{aligned} \tag{1}$$

to describe the optic flow at a given time instant t, where φ_k and η_k are basis functions, and p_k are corresponding motion parameters. As long as the DOF of the motion model increases, the parametrized model describes constant motion, pure translation, affine motion, and so on.

When the motion magnitude is small, the constant intensity assumption [3] is derived by the first order Taylor expansion,

$$I_x u + I_y v + I_t = 0, \tag{2}$$

where I_x, I_y, and I_t are the horizontal, vertical, and temporal partial derivatives of video frame $I(x,y,t)$, respectively.

We assume that the parameter fields of the motion model (1) are sparse in certain domains. Thus, the coefficient $\boldsymbol{g}_k = \Psi_k \boldsymbol{p}_k$ is a sparse signal in the domain Ψ_k, where the transformation can be wavelet, DCT, curvelet, gradient, and so many on. If we have the sparse coefficients g_k, the parameter field can be recovered by $\boldsymbol{p}_k = \Psi_k^+ \boldsymbol{g}_k$, where Ψ_k^+ is the inverse or pseudo inverse of Ψ_k. In this paper, we choose the gradients of motion parameters to apply the sparse regularization for motion estimation.

Employing the prior knowledge about sparsity, the general framework for parametrized motion estimation is formulated as

$$\begin{aligned} \boldsymbol{p}_k^* &= \arg\min_{\boldsymbol{p}_k} \|\mathbf{g}\|_{\ell_0} = \arg\min_{\boldsymbol{p}_k} \sum_k \left\|\Psi_k \boldsymbol{p}_k\right\|_{\ell_0}, \\ \text{s.t.} \quad & I_x u + I_y v + I_t = 0 \quad \forall (x,y), \end{aligned} \tag{3}$$

where $\mathbf{g} = [\boldsymbol{g}_1, \cdots, \boldsymbol{g}_K]$ is the sparse vector field of coefficients. The optic flow is recovered with the optimized motion parameters $\boldsymbol{p}_k^*$ according to the parametrization (1).

According to recent advances in compressive sensing [1, 2], the ℓ_0 problem (3) can be approximately relaxed to a convex ℓ_1 optimization.

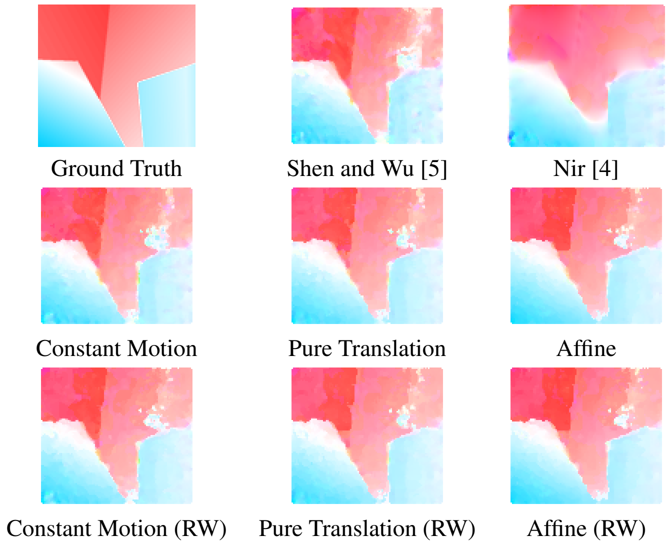

Figure 1: The result optic flow fields of the Venus sequence.

Relaxing the constant intensity assumption by ℓ_1 to obtain more piecewise result [6], we get the ordinary convex optimization model,

$$\boldsymbol{p}_k^* = \arg\min_{\boldsymbol{p}_k} \left\{ \lambda \sum_k \left\| \mathbf{D}_k \boldsymbol{p}_k \right\|_{\ell_1} + \sum_{x,y} \left| I_x u + I_y v + I_t \right| \right\}, \tag{4}$$

where $\mathbf{D}_k$ denotes the discrete gradient operator.

Candès *et al.* proposed a reweighted ℓ_1 optimization strategy to achieve more sparse and accurate solutions [1], so we get the reweighted optimization model,

$$\boldsymbol{p}_k^* = \arg\min_{\boldsymbol{p}_k} \left\{ \lambda \sum_k \left\| \mathbf{W}_k \mathbf{D}_k \boldsymbol{p}_k \right\|_{\ell_1} + \sum_{x,y} \left| I_x u + I_y v + I_t \right| \right\}, \tag{5}$$

where $\mathbf{W}_k$ are the iteratively updated diagonal reweighting matrices.

Experiments certify our arguments that increasing the DOF of motion model enhances the sparsity of motion parameter gradients. So the proposed framework produces robust and accurate motion estimation results. As illustrated by Fig. 1, the pure translation and affine motion models both achieve better results than constant motion. In addition, the reweighted optimization scheme (denoted by RW in Fig. 1) contributes strongly to the improvement of accuracy.

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