Image denoising with patch based PCA: local versus global

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Originally introduced for texture synthesis [5] and image inpainting, patch-based methods have proved to be highly efficient for image denoising. Those methods range from the original Non Local Means (NL-Means) [2], optimal spatial adaptation [6] to the state-of-the-art algorithms BM3D [3], NLSM [8]. Most recent algorithms, either explicitly [1, 7, 8] or implicitly [3], rely on the use of overcomplete dictionaries which are learned from the noisy image or from a larger data set.

While the overcomplete-dictionary-based algorithms exhibit excellent results, they are usually considerably more sophisticated than the traditional algorithms dealing with orthogonal dictionaries. The main question our work responds to is whether or not the improvement achieved with an overcomplete dictionary instead of an orthogonal one is large enough to justify such a sophistication.

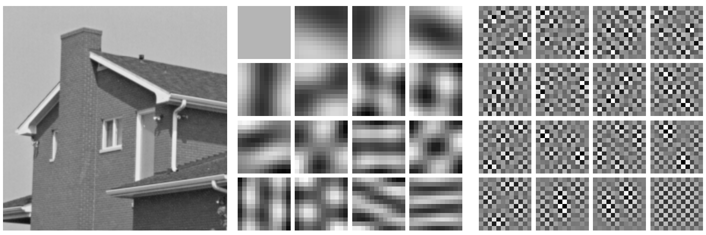
To this end, we investigate in detail the performance of three variants of a simple patch-based denoising algorithm. It consists of the following two steps:

(a) learn an orthogonal basis from the noisy image by performing a Principal Component Analysis (PCA) and decompose the noisy patch in this basis,

(b) obtain the denoised patch by zeroing all the small coefficients in the representation of the noisy patch in the learned basis (ie. using hard thresholding).

This strategy is similar to the wavelet denoising of Donoho and Johnstone [4], with the notable advantage of dealing with an orthonormal basis adapted to the image and computed from noisy patches by PCA In this framework, the set of patches used as input for PCA can be chosen in several ways. We focus on three natural choices: global, hierarchical and local, and carry out a comprehensive empirical study to quantify their differences both in terms of accuracy and computational cost. More precisely, the three variants we consider are:

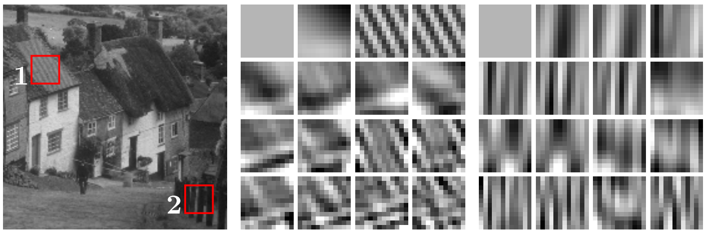
Patch based Global PCA (PGPCA): we create an orthogonal basis that is adapted to the target image by performing a PCA on the whole collection of patches extracted from the noisy image.



(a) Input image (b) 16 first axes (c) 16 last axes

Figure 1: An image (House), its 16 first axes and 16 last axes obtained by a PCA over all the patches of the image.

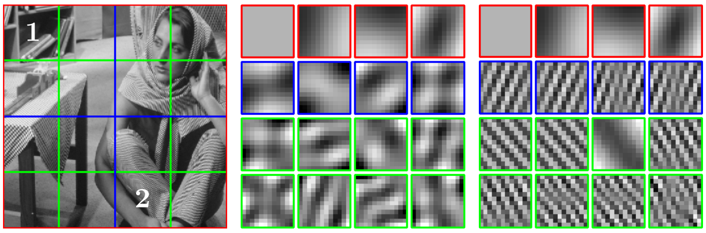
Patch based Hierarchical PCA (PHPCA): we use quadtrees with iterative partitions, *i.e.* we recursively divide the image into four rectangles and proceed to the PCA to the level k of partitioning. At each step a few (usually one) axes are added to the bases and the remaining patches are projected onto the orthogonal supplement of the current orthogonal sub-basis.



(a) Local search windows (b) 16 first axes in window 1 (c) 16 first axes in window 2

Figure 2: An image and its 16 first axes obtained over two stacks extracted respectively in two different local windows. The resulting dictionaries are quite different and suitably describe the local textures of the image.

Patch based Local PCA (PLPCA): we use dynamic localization to build the axes. This strategy relies on a sliding window of size $W_S \times W_S$ in which the patches are selected to proceed to a local PCA [1].



(a) Quadtree decomposition (b) 16 first axes in part 1 (c) 16 first axes in part 2

Figure 3: An image and its 16 first axes obtained over two stacks extracted respectively in two different leaves of the quadtree decomposition. Here, the four main axes are kept at each node of the quadtree and three level of decomposition is used.

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[1] A similar algorithm has been earlier proposed by [9] and [10] in association with Wiener filtering rather than hard thresholding.