Specular Flow and Shape in One Shot

Yair Adato
http://www.cs.bgu.ac.il/~adato
Ohad Ben-Shahar
http://www.cs.bgu.ac.il/~ben-shahar

Department of Computer Science,
Ben-Gurion University of the Negev,
Israel

An image of a purely specular object is just a distortion of its surrounding illumination environment. Therefore, when only little (or nothing) about the illumination environment is known, inferring the geometrical structure of specular objects is a very challenging task. Nevertheless, recent studies have addressed this problem by exploiting relative motion between the observed object, the camera and the environment (e.g.,[1, 4]). In the image plane such motion induces a *specular flow* – the optical flow of observed specular reflections – which has been shown to be independent of environment content and therefore to facilitate shape reconstruction.

The shape-from-specular-flow (SFSF) approach follows the intuitive hierarchical thinking: First, estimate the relevant (specular) flow in the image plane. Next, exploit it for (specular) shape recovery by solving the SFSF equation ([1]). The SFSF equation expresses the relationship between the specular flow, the shape of the specular object, and its motion relative to the environment. A *linear* form for the SFSF equation have been formulated in [4] assuming that the unknown specular object is represented via the field of its reflection vectors, $\mathbf{r} \in S^2$. In this case the SFSF equation becomes

$$\frac{\partial \mathbf{r}(\mathbf{x})}{\partial \mathbf{x}} \mathbf{u}(\mathbf{x}) = \omega \times \mathbf{r}(\mathbf{x}), \tag{1}$$

where $\mathbf{u} = (u, v)$ is the specular flow, and ω is the environment rotation axis. Theoretically, this framework allows dense reconstruction of any general smooth specular object assuming no prior knowledge about the object or the content of the illumination environment.

Unfortunately, however, there are severe practical difficulties to this type of approach. The estimation of specular flow from image sequences is a challenging task (see [2]), putting in question the effectiveness of the shape from specular flow approach as a whole. Moreover, even if the flow is assumed to be known and perfect (that is, without errors), the reconstruction process relies on the solution of partial differential equations that requires enough initial conditions. How to extract these data from the specular flow (or from other image cues) remains an open question.

Simultaneous specular flow and surface estimation

In this work we propose an alternative approach that overcomes many of the problems above. We suggest to couple the estimation of the dense specular flow and the reconstruction of the corresponding specular shape as tightly as possible in the same computational framework. The rational is simple – each structure incorporates constraints that could assist in the estimation of its counterpart, thereby limiting the potential deviation of both from the desired outcome.

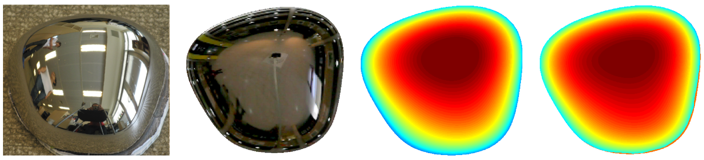
Consider the classical optical flow assumptions for optical flow estimation (e.g., brightness constancy, piecewise smoothness), with additional constraints that emerge from surface geometry considerations (e.g., object smoothness and expected behavior at surface boundaries) and those that reflect the imaging model that links the shape to the specular flow. Formally, we propose to consider an energy minimization problem of the following type

$$\underset{\mathbf{u},\mathbf{r}}{\operatorname{argmin}} E(\mathbf{u},\mathbf{r}) = \underset{\mathbf{u},\mathbf{r}}{\operatorname{argmin}} \int_\Omega E_{flow}(dI, \mathbf{u}) + E_{surface}(\mathbf{u},\mathbf{r}) d\mathbf{x} \tag{2}$$

where Ω is the image domain, dI represents image derivatives and $\mathbf{u}$ and $\mathbf{r}$ are the sought-after specular flow and specular surface, respectively.

The specular flow, being an optical flow, should clearly satisfy the popular brightness constancy assumption (E_{BCA}) and as a regularization, the standard piecewise smoothness constraint (E_{smooth_f}). However, as already discussed recently in [2], the smoothness assumption used so often in the optical flow literature conflicts the nature of the specular flow.

It turns out that much of the problems with the specular flow estimation relate to the magnitude and orientation aspects of the flow. This suggests that perhaps the thinking of the flow in the traditional Cartesian representation is wrong, where a shift in representation may solve some of the problems. Recently, we have advocated such a shift in the representation of motion from Cartesian to the polar coordinate system [3] and have demonstrated its advantages especially when the estimated motion vector field is relatively complex, as indeed is the case with specular flows. In the full paper we suggest an enhanced one shot algorithm in which the specular flow is represented using polar coordinates.



(a) (b) (c) (d)

Figure 1: Evaluation of the basic algorithm. **(a)** A real specular object with ground truth data from [2]. **(c)** A sample frame observed from the object. **(c)** Ground truth shape data. **(d)** Estimate shape using our basic algorithm.

In order to formulate the constraints between the unknown surface and flow, one should choose an imaging model. Here we follow the model proposed in the SFSF literature (e.g. [1, 4]), to enjoy the advantages of the *linear* SFSF equation (Eq. 1). Using this representation, the surface constraint is derived by manipulating Eq. 1:

$$\begin{aligned} E_{ref} &= \psi((\nabla r^a \cdot \mathbf{u} - \omega_3 r^b + \omega_2 r^c)^2) + \psi((\nabla r^b \cdot \mathbf{u} + \omega_3 r^a - \omega_1 r^c)^2) \\ &+ \psi((\nabla r^c \cdot \mathbf{u} - \omega_2 r^a + \omega_1 r^b)^2), \end{aligned} \tag{3}$$

where $\omega = (\omega_1, \omega_2, \omega_3)$ is the axis of rotation, $\mathbf{r} = (r^a, r^b, r^c)$ is the reflection vector field expressed by its three coordinates at each point and ψ robust function.

Next, we apply similar regularization to the surface by minimizing either its first or second derivatives $E_{smooth_r} = \psi(\|\Delta \mathbf{r}\|^2)$. We also remember that $\mathbf{r}$ must be unit length, therefore we define $E_{coher_r} = (\|\mathbf{r}\|^2 - 1)^2$.

Finally, the surface boundaries can also be exploited since the geometry of specular reflection dictates that under orthographic projection the reflection vectors along the occluding boundary must be $(0, 0, -1)$. Putting it all together we obtain the following problem

$$\begin{aligned} \underset{\mathbf{u},\mathbf{r}}{\operatorname{argmin}} E(\mathbf{u},\mathbf{r}) &= \underset{\mathbf{u},\mathbf{r}}{\operatorname{argmin}} \int_\Omega E_{BCA} + \alpha_1 E_{smooth_f} + \beta(E_{ref} + \alpha_2 \, E_{smooth_r}) + E_{coher_r} d\mathbf{x} \\ s.t. \quad \mathbf{r}(\mathbf{x}) &= (0,0,-1) \qquad \forall \mathbf{x} \in \partial \mathbf{r} \,. \end{aligned} \tag{4}$$

In this equation, the parameters α_i control how deep would be the flow and surface regularizations while β represents the confident in the reconstructed surface.

Solving Eq. 4 is the heart of our proposed approach. Although this appears to be a complicated exercise, we demonstrate in the full paper how it can be done in ways similar to previous methods from the optical flow literature with complexity different only by a constant factor.

To conclude, we propose a new approach that can estimate specular flow and recover the specular shape that gave rise to it all in one shot. Given how all previous shape-from-specular-flow approaches require an estimation of specular flow, which present optical flow algorithm fail to reasonably prove, our suggested approach is the first one to facilitate specular shape reconstruction from real image sequences. Contrary to all previous approaches, initial shape conditions are not needed either.

Considering future work, one could consider better and more elaborate regularizations that address the structure of parabolic singularities more explicitly. Furthermore, it is a practical challenge to consider the estimation of the rotation axis also as part of the optimization process, perhaps at the price of increased ambiguity.

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