Geometric Potential Force for the Deformable Model

Si Yong Yeo[1]
465186@swansea.ac.uk

Xianghua Xie[2]
x.xie@swansea.ac.uk

Igor Sazonov[1]
i.sazonov@swansea.ac.uk

Perumal Nithiarasu[1]
p.nithiarasu@swansea.ac.uk

[1] School of Engineering,
Swansea University, UK

[2] Department of Computer Science,
Swansea University, UK

Deformable models often vary in object boundary representation and the external force field used. Conventional methods, i.e. geodesic active contour model [1], have difficulties in dealing with boundary concavities, weak edges and image noise. Some approaches such as the generalized gradient vector flow (GGVF) [3] showed some improvements but have convergence issues caused by saddle or stationary points in its force field. Very recently, Xie and Mirmehdi [2] proposed a novel edge based model which showed significant improvements in handling weak edges and complex geometries. However, its analogy based on magnetostatics can not be directly applied to 3D or higher dimensional space.

In this work, we provide the generalization of the MAC model [2] and define the external force field based on hypothesized geometrically induced interactions between the relative geometries of the deformable model and the object boundaries (characterized by image gradients). In other words, the magnitude and direction of the interaction forces are based on the relative position and orientation between the geometries of the deformable model and image object boundaries, and hence, it is called the *geometric potential force (GPF)* field. The bidirectionality of the new external force field can facilitate arbitrary cross-boundary initialization, which is a very useful feature to have, especially in the segmentation of complex geometries, and in handling weak edges. Similar to MAC, the proposed external force field is dynamic in nature as it changes according to the relative position and orientation between the evolving deformable model and object boundary.

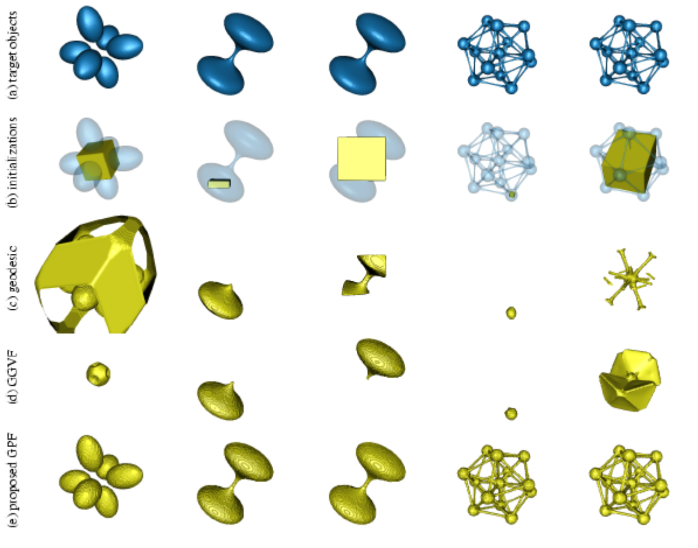

Figure 1: Shape recovery from synthetic images: (a) isosurfaces of various shapes to be recovered from synthetic images, (b) initial deformable models (yellow) with input shapes (blue, semi-transparent), (c) recovered shape using geodesic, (d) GGVF, (e) proposed GPF

Consider two elements dA_1 and dA_2 on two contours/surfaces, with unit normals $\hat{n}_1$ and $\hat{n}_2$ respectively. The hypothesized interaction force dF acting on dA_1 due to dA_2 is given as

$$dF = dA_1\,\hat{n}_1\,dG, \qquad dG = \frac{dA_2}{r_{12}^k}\,(\hat{r}_{12}\cdot\hat{n}_2) \tag{1}$$

where dG is considered as the geometrically induced potential created by element dA_1. Here, r_{12} is the distance between dA_1 and dA_2, and $\hat{r}_{12}$ is the unit vector pointing from dA_1 to dA_2; k is a positive constant that affects the magnitude of the interaction force based on the distance between dA_1 and dA_2, and is set to the same as the dimension of the image data.

Figure 2: Shape recovery from weak edges.

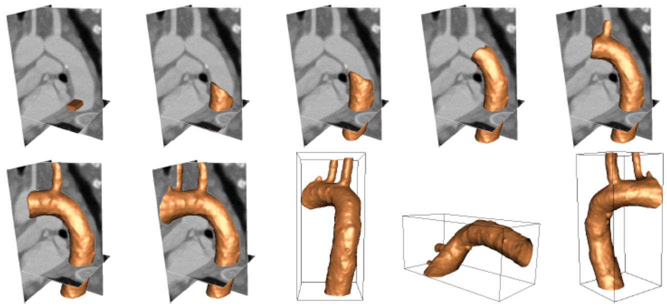

Figure 3: Segmentation of human aorta using the GPF deformable model.

Let $u(x)$ be the 3D image, whereas dA_1 and dA_2 belong to the deformable surface and object boundary respectively. We compute the total geometric potential field strength $G(x)$ at every voxel x given as:

$$G(x) = \sum_{y\,\in\,S,\,y\neq x} \frac{\hat{r}_{12}}{r_{12}^k}\cdot\hat{n}_2(y)\,dA_2 \tag{2}$$

where S denote the set containing all the edge voxels, and y denote a boundary voxel. $\hat{r}_{12}$ is the unit vector from x to y, and r_{12} is the distance between them. Computation of (2) is based on the 3D FFT. The force acting due to the geometrically induced potential field on the deformable surface $\mathscr{C}$ at the position $x \in \mathscr{C}$ can then be given as:

$$F(x) = dA_1\,\hat{n}(x)\,G(x) \tag{3}$$

The evolution of the deformable model $C(x,t)$ under the force field $F(x)$ can be given as: $C_t = \left(F\cdot\hat{n}\right)\hat{n}$. The complete geometric potential deformable model evolution with mean curvature flow is formulated as:

$$C_t = \alpha\,g(x)\,\kappa\,\hat{n} + (1-\alpha)(F\cdot\hat{n})\,\hat{n} \tag{4}$$

where α is a weighting parameter, $g = \frac{1}{1+|\nabla u|}$ is the stopping function and κ is the mean curvature.

The proposed GPF deformable model can handle cross-boundary initializations, and resolve saddle and stationary points issues due to its bidirectionality. In addition, the new vector force field is dynamic in nature, and can attract the deformable model into highly concave regions, and propagate through long thin structures. The comparative study on various geometries showed significant improvements in convergence capability and initialization invariancy on existing state-of-the-art methods.

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