NURBS Warps

Florent Brunet[123]
florent.brunet@univ-bpclermont.fr
Adrien Bartoli[1]
adrien.bartoli@gmail.com
Nassir Navab[2]
navab@cs.tum.edu
Rémy Malgouyres[3]
remy.malgouyres@laic.u-clermont1.fr

[1] LASMEA
CNRS/Université Blaise Pascal
Clermont-Ferrand, France
[2] CAMPAR
Technische Universität München
Garching bei München, Germany
[3] LAIC
Université d'Auvergne
Clermont-Ferrand, France

Given two images of a threedimensional surface, a warp is an $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ function that maps a point from the first image to the corresponding point in the second image (see figure 1). For example, two images of a rigid scene taken by a perspective camera that rotates around its optical center are related to each other by an homographic warp. In the presence of deformable surfaces, and for an arbitrary camera displacement, the warp is more complex. Several parametric warps were proposed and proved efficient, such as the Thin-Plate Spline warp [1, 2] and the B-Spline warp (hereinafter abbreviated *BS-Warp* and also called Free-Form Deformation [3]).

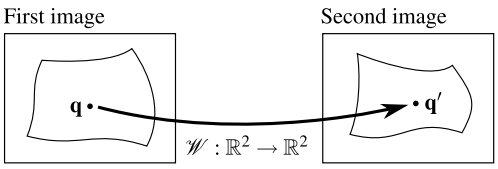

Figure 1: A warp $\mathscr{W}$ is a function that maps a 2D point $\mathbf{q}$ from the first image to a point $\mathbf{q}'$ in the second image.

In this paper, we bring several contributions. We first demonstrate that the classical BS-Warp corresponds to affine imaging conditions, in the sense that it models the affine projection of some 3D surface. We then propose our most important contribution: a novel parametric warp we call *NURBS-Warp*, that extends the classical BS-Warp to perspective projection. This warp has a simple analytical form: it is obtained as the two-way tensor-product of bivalued Non-Uniform Rational B-Splines (NURBS). Finally, we give algorithms for the feature-based estimation of our NURBS-Warp.

Our NURBS-Warps are compared to standard BS-Warps for both synthetic and real images. These experiments show that our NURBS-Warp gives better results than the other warps, especially when the perspective effects are significant. We also show that the benefit brought by our new warp is more sensible when the observed surface deformations are limited. However, when the surface deformations are important, both the NURBS-Warps and the BS-Warps require a lot of control points. Since it increases the number of degrees of freedom, the BS-Warp can then also cope with perspective effects.

BS-Warps. If we consider that the observed surface is modeled by a threedimensional tensor-product B-Spline, the BS-Warp corresponds to the transformation between the two images under affine imaging conditions (see figure 2 for an illustration). Besides, we experimentally show in the article that BS-Warps behave poorly for perspective imaging conditions.

NURBS-Warps. We build a new warp we call *NURBS-Warp*. As illustrated in figure 3, it is obtained by replacing the affine projection by the perspective one. Experimental results show that these new warps are particularly well suited when the perspective effects are large. An example is given in figure 4.

Parameter estimation. Since the BS-Warp is analytically linear, there exists a closed-form solution to estimate its parameters from point correspondences. The NURBS-Warp is not linear. However, it is possible to estimate its parameters using an iterative algorithm such as Levenberg-Marquardt. We propose three approaches to initialize this algorithm:

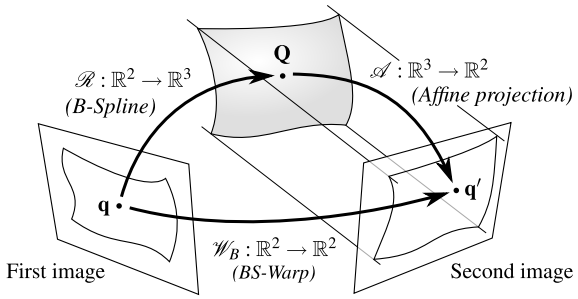

Figure 2: A BS-Warp can be seen as the result of a threedimensional B-Spline surface projected under affine conditions.

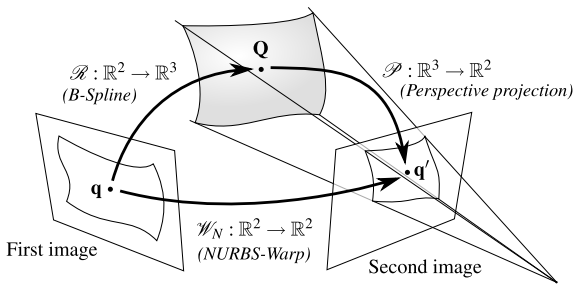

Figure 3: A NURBS-Warp can be seen as the result of a threedimensional B-Spline surface projected under perspective conditions.

- act as if the images were taken under affine imaging conditions;
- act as if the warp relating the two images was an homography;
- use an algebraic approximation to the transfer function.

[1] A. Bartoli, M. Perriollat, and S. Chambon. Generalized Thin-Plate Spline warps. In *Proceedings of the 7th Conference on Computer Vision and Pattern Recognition*, pages 1–8, 2007.

[2] F. L. Bookstein. Principal warps: Thin-Plate Splines and the decomposition of deformations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 11(6):567–585, June 1989.

[3] D. Rueckert, L. I. Sonoda, C. Hayes, D. L. G. Hill, M. O. Leach, and D. J. Hawkes. Nonrigid registation using Free-Form Deformations: Application to breast MR images. *IEEE Transactions on Medical Imaging*, 18(8):712–721, August 1999.

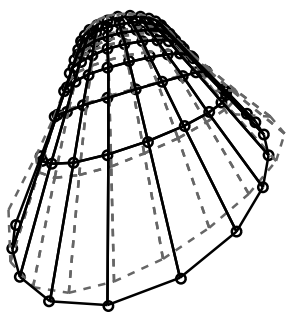

Figure 4: The NURBS-Warp (black visualization grid) better models the perspective effect than the BS-Warp (dashed grid). The black circles represent the ground truth location of the vertices.