

# Comparing Variants of the Beckmann Model

Hossein Ragheb<sup>1</sup> and Edwin R. Hancock<sup>2</sup>

<sup>1</sup>Department of Computer Engineering, Bu-Ali Sina University,  
Hamedan, Iran, PO Box 65175-4161, ragheb@basu.ac.ir

<sup>2</sup>Department of Computer Science, University of York, York,  
YO1 5DD, UK., erh@cs.york.ac.uk

## Abstract

Recent work has shown how the Fresnel coefficient can be used to improve the predictions of the Beckmann-Kirchhoff (BK) model for moderately-rough surfaces. This paper aims to compare the predictions of a number of alternative models including those of Lafortune et al. (LFTG) and Oren-Nayar (ON) against the BRDF measurements from the CURET database. Excluding the specular reflectance component, we explore how these models account for the diffuse reflectance component. Our main conclusions are as follows. First, for moderately rough samples Fresnel correction provides the most accurate predictions for the majority of the samples studied. Second, although there are several surface samples in the CURET database for which the ON model provides the best fit, the ON model and the LFTG model fail to fit the data well for most of the surface samples. Third, for all surface samples studied, the Fresnel corrected variant of the BK model gives more realistic predictions than the empirical reflectance models such as the LFTG model.

## 1 Introduction

The modelling of rough surface reflectance is important in both computer vision and computer graphics, and has been the subject of sustained research activity for some four decades. For very-rough surfaces, one approach is to use a model which describes the distribution of surface wall cavity angles [1]. For rough surfaces which present a shiny appearance, roughness can be modelled using the angular distribution of microfacets [2]. An alternative that can capture both effects is to describe the roughness phenomenon using the variance and the correlation of variations in the surface height distribution [3]. Although both purely empirical approach [4] have been used in graphics and semi-empirical ones [1, 5] used in computer vision, one of the most principled methods is to use wave scattering theory [6] to understand the interaction of light with the surface relief distribution. According to this latter approach, the Kirchhoff integral is used to account for the scattering of light from rough surfaces characterized using the variance and correlation length of the surface relief distribution. The model is mathematically quite complex, and is hence not well suited for analysis tasks of the type encountered in computer vision. However, He et al. [3] have developed a comprehensive model that incorporates complex factors including surface statistics, sub-layer scattering, and polarization. Also, Stam [7] has applied the Kirchhoff approach to include diffraction effects in surface rendering.

Unfortunately, the Beckmann-Kirchhoff (BK) model fails to account for the observed radiance at large scatter angles due to energy absorption, self shadowing and multiple scattering effects. Some of the problems have recently been overcome by Vernold and Harvey [8] who have used a simple Lambertian form factor to modify Beckmann's predictions. There is considerable debate about the meaning and proper use of the form

factor, and so, a number of alternatives have been used in the wave scattering literature [9, 10]. However, despite these advances the BK model provides closed-form solutions only for slightly-rough and very-rough surfaces.

The aim in this paper is to fill this gap in the literature by developing an extension of the BK model that can be applied to surfaces of intermediate roughness. To do this we borrow ideas from the semi-empirical modelling of rough surface reflectance and incorporate subsurface refractive attenuation of light prior to wave scattering. This effect can be accommodated by multiplying the Kirchhoff scattering kernel by a form factor that depends on the Fresnel coefficient. We use a correction term, similar to that used by Wolff et al. [5], in place of the Lambertian term of Vernold and Harvey. We can also compare our approach to that introduced by Jensen et al. for computer graphics [11]. Their model for subsurface transport in translucent materials is derived by multiplying the diffuse component corresponding to multiple scattering by two Fresnel terms (for incoming light and outgoing radiance). Finally, we compare the new model with existing models using the BRDF data from the CURET database [12]. The pictures of the surface samples studied are shown in Fig. 1. The new model outperforms the alternatives studied and provides remarkably good fits to the CURET data for surfaces of intermediate roughness.

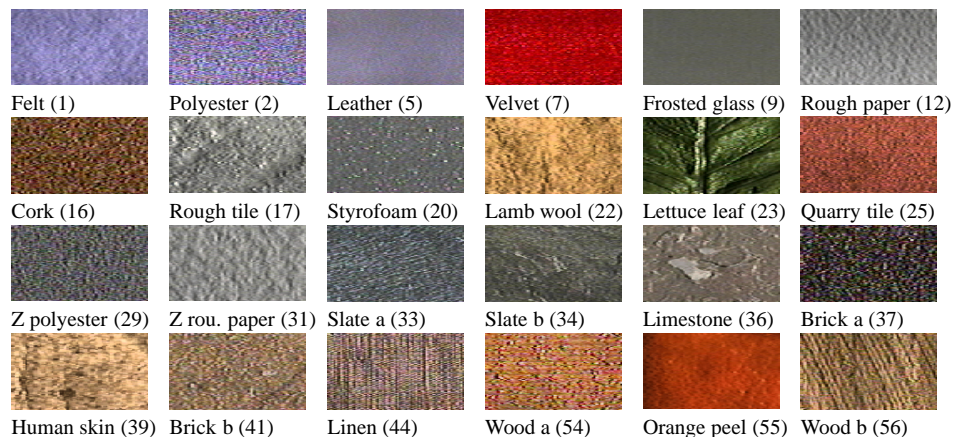


Figure 1: Surface samples studied with their index number as appear in the CURET database.

## 2 Classical and New Variants of the Beckmann Model

The Beckmann-Kirchhoff (BK) theory attempts to account for the wave-interactions of light with rough surfaces. The starting point is Kirchhoff’s integral which gives the scattered light-field amplitude. Beckmann’s contribution was to show how to apply the theory to rough surfaces, and how to obtain simplifications to the Kirchhoff integral under different roughness conditions. The details of the theory are comprehensively described in the monographs of Beckmann and Spizzichino [6], and Ogilvy [10].

*Geometry and Radiometry:* We are interested in a surface illuminated by a parallel beam of light of known wavelength  $\lambda$  and viewed by a camera which is sufficiently distant from the surface so that perspective effects may be ignored. The incident light-beam has direction-vector  $\vec{S}$  and the surface normal is  $\vec{N}$  (Fig. 2.a). The direction of the optic axis of the camera, or viewing direction, is denoted by  $\vec{V}$ . The incident light has zenith angle  $\theta_i$  and azimuth angle  $\phi_i$  with respect to the surface normal, while the zenith and azimuth angles of the viewer (scattering) direction with respect to  $\vec{N}$  are  $\theta_s$  and  $\phi_s$ . The radiance of

the incident light-beam at the location on the surface is  $L_i(\theta_i, \phi_i)$  and the outgoing radiance is  $L_o(\theta_i, \phi_i, \theta_s, \phi_s)$ . If  $v(\theta_i, \phi_i, \theta_s, \phi_s)$  is the bidirectional reflectance distribution function (BRDF) for the surface, we can write  $L_o(\theta_i, \phi_i, \theta_s, \phi_s) = v(\theta_i, \phi_i, \theta_s, \phi_s)L_i(\theta_i, \phi_i) \cos \theta_i d\omega$  where  $d\omega$  is the small solid angle of the source subtended at the surface. We will make use of this relationship in Section 3 when we compare the outgoing radiance predictions by different models with BRDF data from the CURET database [12]. In the CURET database, the BRDF data have been tabulated by assuming that each surface radiance value is related to the corresponding image pixel brightness value by a gain and an offset. For each surface sample, we normalize the outgoing radiance values by dividing them by the maximum radiance value. This ensures that the comparisons are accurate.

In Fig. 2.a we show the scattering geometry on the tangent plane to the imaged surface. We align the local coordinate system  $(X_0, Y_0, Z_0)$  so that the projection of the light vector onto the tangent plane points in the direction of the negative  $X_0$  axis. Hence,  $\phi_i = \pi$  for all scattering geometries. This configuration is identical to that used by Beckmann.

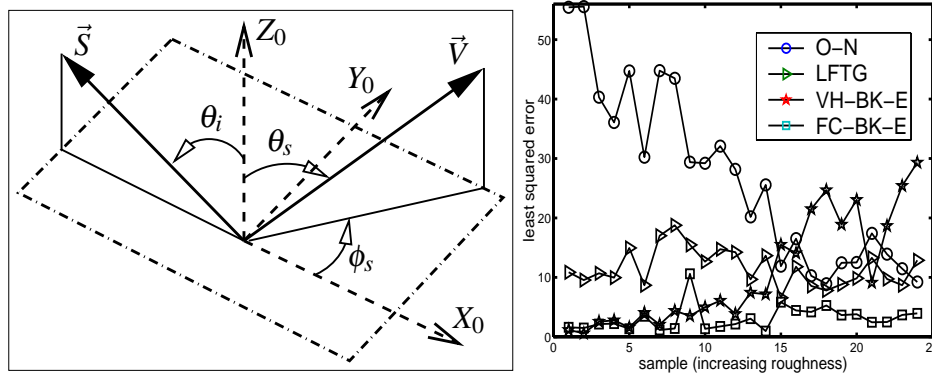


Figure 2: Left (a): scattering geometry on the tangent plane coordinate system: the incident light propagates in the direction  $\vec{S}$ , while the direction for the scattered light is  $\vec{V}$ ; right (b): least-squared-error (LSE) for the models studied versus sample index sorted according to increasing surface slope estimates corresponding to the FC-E model.

*Original Model:* The BK model has two main physical parameters. The first of these is the root-mean-square height deviation of the topographic surface features about the mean surface level which is denoted by  $\sigma$ . The height variations  $\Delta z$  of the surface are assumed to follow the Gaussian distribution function  $W(\Delta z) = (1/\sigma\sqrt{2}) \exp(-\Delta z^2/2\sigma^2)$ . The second parameter is the correlation length  $T$  which is defined in terms of the surface correlation function  $C(\tau)$ .  $T$  characterizes the random nature of a surface profile and the relative spacing of peaks and valleys. Specifically, the correlation length is the lag-length at which the Gaussian correlation function  $C(\tau) = \exp(-\tau^2/T^2)$  drops to  $1/e$ . Similarly, for the exponential correlation function  $C(\tau) = \exp(-|\tau|/T)$  it follows that  $C(T) = 1/e$ .

We interpret Beckmann's result as scattered radiance  $L_o(\theta_i, \phi_i = \pi, \theta_s, \phi_s)$ . According to the BK model [6], for different roughness regimes there are approximate forms for the diffuse scattering component. For very-rough surfaces there are two interesting formulas that arise with different correlation functions. When the correlation function is *Gaussian* [6], then with the geometry outlined in Fig. 2.a, the diffuse scattered radiance is

$$L_o^{BK-G}(\theta_i, \pi, \theta_s, \phi_s, \frac{\sigma}{T}) = \frac{\pi T^2}{A \sigma^2} \frac{F_{BK}^2(\theta_i, \theta_s, \phi_s)}{v_z^2(\theta_i, \theta_s)} \exp[-T^2 v_{xy}^2(\theta_i, \theta_s, \phi_s) / 4 \sigma^2 v_z^2(\theta_i, \theta_s)] \quad (1)$$

Alternatively, if the correlation function is *exponential* [13] then we have

$$L_o^{BK-E}(\theta_i, \pi, \theta_s, \phi_s, \frac{\sigma}{T}) = \frac{2\pi T^2 F_{BK}^2(\theta_i, \theta_s, \phi_s)}{A\sigma^2 v_z^2(\theta_i, \theta_s)} [1 + T^2 v_{xy}^2(\theta_i, \theta_s, \phi_s) / \sigma^2 v_z^2(\theta_i, \theta_s)]^{(-3/2)} \quad (2)$$

where  $v_x(\theta_i, \theta_s, \phi_s) = k(\sin \theta_i - \sin \theta_s \cos \phi_s)$ ,  $v_y(\theta_s, \phi_s) = -k(\sin \theta_s \sin \phi_s)$ ,  $v_z(\theta_i, \theta_s) = -k(\cos \theta_i + \cos \theta_s)$ ,  $v_{xy}^2(\theta_i, \theta_s, \phi_s) = v_x^2(\theta_i, \theta_s, \phi_s) + v_y^2(\theta_s, \phi_s)$  and  $k = 2\pi/\lambda$ . The scattering takes place from a rectangular surface patch of area  $A$ . In practice, the single parameter  $\sigma/T$  (surface slope) controls the model predictions. Note that no closed-form solution has been derived for moderately-rough surfaces. Finally, the choice of the *geometric factor*  $F$  is of critical importance to the model [9]. The Beckmann's geometric factor  $F_{BK}$  is

$$F_{BK}(\theta_i, \theta_s, \phi_s) = (1 + \cos \theta_i \cos \theta_s - \sin \theta_i \sin \theta_s \cos \phi_s) / [\cos \theta_i (\cos \theta_i + \cos \theta_s)] \quad (3)$$

*Vernold-Harvey (VH) Modification:* Unfortunately, as highlighted by several authors [8, 10], Beckmann's formula for the geometric factor  $F$  fails to reliably predict the scattering behavior at large angles of incidence and scattering. Hence, there is considerable debate about the meaning and proper use of this factor [3, 7, 8, 9]. In fact, a variety of forms for  $F$  have been proposed in the wave scattering literature. For instance, based on phenomenological arguments, Vernold and Harvey [8] argue for the use of the geometrical term  $F_{VH}^2 = \cos \theta_i$  that is Lambertian in form and depends only on the cosine of the incidence angle. This modification gives reasonable experimental agreement with scattering data at large angles of incidence and scattering. In what follows we propose a Fresnel corrected variant of the BK model by suggesting an alternative form factor.

*Attenuation Factor:* The Fresnel coefficient has been widely used to account for subsurface scattering. For instance, Torrance and Sparrow [2] have included the Fresnel term in their model for off-specular intensity. It is also used in the more complex reflectance model of He et al. [3] which attempts to account for a number of effects including subsurface scattering. Wolff [5] has developed a physically motivated model for diffuse reflectance from smooth dielectric surfaces. The model accounts for subsurface light scattering using the Fresnel coefficient ( $0 \leq f(\alpha_i, r) \leq 1.0$ ) which is given by

$$f(\alpha_i, r) = [\sin^2(\alpha_i - \alpha_t) / \sin^2(\alpha_i + \alpha_t)] [1 + \cos^2(\alpha_i + \alpha_t) / \cos^2(\alpha_i - \alpha_t)] \quad (4)$$

where the transmission angle of light into the dielectric surface  $\alpha_t$  is given by Snell's law:

$$r = (\sin \alpha_i) / (\sin \alpha_t) \Rightarrow \alpha_t = \sin^{-1}[(\sin \alpha_i) / r] \quad (5)$$

where the parameter  $n$  is the index of refraction of the dielectric medium. When light is transmitted from air into a dielectric  $r = n$  and  $\alpha_i = \theta_i$ . However, when transmission is from a dielectric into air, then  $r = 1/n$  and  $\alpha_i = \sin^{-1}[(\sin \theta_s) / n]$ . Almost all commonly found dielectric materials have an index of refraction,  $n$ , in the range [1.4, 2.0]. As a result the Fresnel function is weakly dependent upon the index of refraction for most dielectrics.

*Fresnel Correction (FC) Variant:* Our approach is similar to that followed by Wolff et al. [5] which combines the Fresnel term with the ON model to develop a model for moderately-rough surfaces. We exploit the Fresnel model used by Wolff, which includes the effects of both incidence and reflectance angles. Specifically, the geometrical term  $F^2$  in the BK model variant for very-rough surfaces is replaced by a Fresnel correction term to produce a Fresnel corrected BK model. Whereas Vernold and Harvey [8] have

replaced the  $F^2$  term by  $\cos \theta_i$ , we replace it by the factor  $\cos \theta_i$  multiplied by two Fresnel terms (for incidence and for reflection). Hence, we aim to improve the VH modification by considering the subsurface scattering effects on the scattered radiance.

Since the Kirchhoff theory is a single scatter model [10], neither multiple scattering nor subsurface multiple scattering are taken into account. Our proposed model for moderately-rough dielectrics combines both subsurface scattering and multiple scattering from surface irregularities. The Fresnel correction term accounts for subsurface multiple scattering, while the VH model is used to account partially for multiple scattering from surface irregularities. When combined, these two models lead to a new variant of the BK theory which gives improved predictions for the scattered radiance. Our corrected model variant uses this multiplicative approach. Specifically, the Fresnel correction term is

$$F_{FC}^2(\theta_i, \theta_s, n) = [1 - f(\theta_i, n)] \{1 - f(\sin^{-1}[(\sin \theta_s)/n], 1/n)\} \cos \theta_i \quad (6)$$

### 3 Testing Models Against Data

In this section we compare the predictions of different models with BRDF data for surface samples from the CURET database [12]. The samples studied (with their index numbers) are shown in Fig. 1. We have chosen 24 moderately-rough surface samples, selected using estimates of the surface slope parameter [14]. The variants of the BK model studied here are the VH modification and our FC variant. We are only interested in the model variants that apply to moderately-rough surfaces, and where the correlation functions are either Gaussian (G) or exponential (E). Table 1 lists a set of acronyms used to refer to these models in Tables 2-3 and Figs. 3-8. As an empirical reflectance model, we choose the Lafortune et al. (LFTG) model [4] which is an improved version of the Phong model. When only the diffuse reflectance component is considered, the surface radiance is

$$L_o^{LFTG}(\theta_i, \theta_s, n_d) = [\rho(n_d + 2)/2\pi](\cos \theta_i \cos \theta_s)^{n_d} \quad (7)$$

In Eq. (7),  $\rho$  is the surface albedo and the exponent  $n_d$  is the single parameter controlling the fit of the model to the data. We also investigate the predictions of a variant of the ON model which includes interreflections [1]. For the ON model, we use the parameters which have been estimated and tabulated in the database for each sample. For the BK model variants and the LFTG model, on the other hand, there are no such parameter tabulations for the samples. Hence we find the parameters using a least-square method which involves exhaustively searching for the parameters which give the best fit of the relevant model to the BRDF data. To compare theory with experiment, we convert the BRDF measurements  $v(\theta_i, \phi_i, \theta_s, \phi_s)$  in the tables into the normalized outgoing radiance  $L_o(\theta_i, \phi_i, \theta_s, \phi_s)$  estimates. This is a straightforward task since the incidence angle  $\theta_i$  and the brightness of the light-source  $L_i$  which is constant for each surface sample are tabulated, and hence  $L_o(\theta_i, \phi_i, \theta_s, \phi_s) = v(\theta_i, \phi_i, \theta_s, \phi_s)L_i \cos \theta_i d\omega$ . This formula offers the advantage that it avoids problems due to singularities in the BRDF that may occur at grazing incidence angles. Since here we use normalized radiance values for the purpose of comparing model predictions with the data measurements, we need to know neither the constant value  $L_i$  nor the constant value  $d\omega$  for each surface sample.

Specifically, we seek the model parameter values that minimize the mean squared error  $\sum_{k=1}^K [L_o^M(\theta_i^k, \phi_i^k, \theta_s^k, \phi_s^k) - L_o^D(\theta_i^k, \phi_i^k, \theta_s^k, \phi_s^k)]^2$ , where  $L_o^M$  is the normalized radiance value predicted by the model,  $L_o^D$  that obtained from the BRDF data, and  $k$  runs over the index number of the illumination configurations used. We use the  $K = 198$  of the 205 configurations provided in the CURET database; these correspond to the non-specular directions, and hence only the diffuse radiance needs to be taken into account in the model

fitting. To locate the least-squares model parameter values, we test 500 equally spaced values of the surface slope parameter  $\sigma/T$  in the interval [0.1, 5.1] for the BK model variants, and 500 values in the interval [0.1, 20.1] for the LFTG model parameter,  $n_d$ . The values of the best-fit parameters together with their least squared errors (LSE) are shown in Tables 2 and 3, respectively. Note that for each surface sample, we use the known value of the refractive index  $n$  and we do not attempt to vary its value as a fitting parameter.

Table 1: The improved variants of the BK models studied and the acronyms used.

model variant	correlation fun.	acronym	scattered radiance
Vernold-Harvey	Gaussian	VH-G	$L_o^{VH-G} = L_o^{BK-G} \times \cos \theta_i / F_{BK}^2$
Vernold-Harvey	Exponential	VH-E	$L_o^{VH-E} = L_o^{BK-E} \times \cos \theta_i / F_{BK}^2$
Fresnel Correction	Gaussian	FC-G	$L_o^{FC-G} = L_o^{BK-G} \times F_{FC}^2 / F_{BK}^2$
Fresnel Correction	Exponential	FC-E	$L_o^{FC-E} = L_o^{BK-E} \times F_{FC}^2 / F_{BK}^2$

In Fig. 2.b we summarize the results of performing least-squares fits to the BRDF data. Here we have sorted the samples according to increasing slope parameter values obtained using the FC-E variant. For each model we have plotted the LSE values as a function of surface sample numbers. The plot is for the variants of the BK model with the exponential correlation function (which is similar to that with the Gaussian correlation function). There are a number of features that drawn from the plot. First, the LSE for the ON model decreases with increasing roughness, i.e. the fit of the model to different data samples improves with increasing roughness. For the VH modification, on the other hand, the LSE increases with increasing roughness. With this model, the best fits are obtained to velvet (7), slate b (34) and brick b (41). The LFTG model is relatively insensitive to the roughness order. Its LSE is lower than that obtained by the ON model and higher than that obtained with the VH modification for most of the samples. For the FC variant, the LSE is consistently lower than all of the alternatives. The improvements over the alternatives are most marked for the moderately-rough samples, i.e. slate a (33), skin (39), linen (44) and wood a (54). Hence, the model is a better fit to data over all scales of roughness.

In Figs. 3-6, for each sample we show scatter plots of the predicted normalized radiance as a function of the measured normalized radiance for the models investigated. The better the data is clustered around the diagonal straight line, the better the agreement between experiment and theory. We have shown only the plots corresponding to the ON model and the LFTG model together with those for the VH-E and FC-E model variants. Additional experiments, not included here, show that these variants of the BK model result in the best fits with the data when compared to the remaining alternatives. First we consider the behavior of the ON model. From Figs. 3-6 it is clear that the model gives the best results for some very-rough surface samples, e.g. felt (1), polyester (2), rough paper (12), and brick a (37). From the scatter plots, the fits appear qualitatively good, except perhaps at large radiance values. However, for the smooth and moderately rough samples, the method does not perform well when compared with the alternatives. For both the FC and VH variants of the BK model, it is the exponential correlation function that seems to give a better qualitative fit to the data than the Gaussian correlation function. The main reason for the poor fit of the LFTG model to the data is that it gives identical radiance values for the configurations with identical zenith angles and different azimuth angles.

## 4 Conclusions

We have explored how to extend the Beckmann's (BK) model [6] to surfaces of intermediate roughness. To investigate the utility of the new model, we have experimented

with BRDF data from the CURET database [12] for a wide range of surface samples. The results show that the new model gives good agreement with the data for most illumination configurations. The model gives the best fits to the data for moderately-rough surfaces. The model outperforms the alternatives studied, specifically the empirical reflectance model of Lafortune et al. [4]. The scope of our future research is to compare the new model with purely statistical reflectance models [15]. Moreover, studying the model against recently released BRDF measurements by Matusik et al. [16] is another priority.

Table 2: Model parameter estimates corresponding to the least squared errors of their fit to the data; the parameters of the ON model are identical to those given in the CURET database.

Surface sample (no)	VH-G	VH-E	FC-G	FC-E	ON( $\rho$ )	ON( $\sigma$ )	LFTG
Felt (1)	1.85	3.84	1.10	2.34	0.387	0.415	0.62
Polyester (2)	2.23	4.85	1.28	2.78	0.380	0.577	0.50
Leather (5)	1.41	2.75	0.78	1.58	0.580	0.180	0.86
Velvet (7)	4.94	5.10	0.56	1.17	0.171	0.751	1.90
Frosted glass (9)	0.57	1.25	0.45	0.90	0.142	0.416	2.66
Rough paper (12)	1.59	3.16	1.02	2.16	0.645	0.311	0.62
Cork (16)	1.48	2.93	0.71	1.42	0.249	0.660	1.26
Rough tile (17)	1.00	1.77	0.46	0.89	0.508	0.204	1.78
Styrofoam (20)	2.12	4.53	1.15	2.45	0.477	0.510	0.62
Lambs wool (22)	2.51	5.10	1.19	2.35	0.325	0.978	0.78
Lettuce leaf (23)	0.70	0.99	0.32	0.63	0.179	0.242	3.70
Quarry tile (25)	0.56	1.06	0.39	0.77	0.106	0.361	3.50
Z polyester (29)	1.93	4.06	1.10	2.32	0.405	0.523	0.66
Z rough paper (31)	1.55	3.05	0.97	2.06	0.567	0.318	0.66
Slate a (33)	0.56	1.24	0.47	0.94	0.030	0.357	1.94
Slate b (34)	0.38	0.77	0.25	0.47	0.068	0.310	9.50
Limestone (36)	1.45	2.89	0.75	1.51	0.293	0.414	1.06
Brick a (37)	4.75	5.10	2.25	4.80	0.133	0.893	0.38
Human skin (39)	1.28	2.27	0.67	1.34	0.199	0.579	1.22
Brick b (41)	0.37	0.76	0.25	0.47	0.058	0.276	9.06
Linen (44)	1.35	1.26	0.74	1.52	0.252	0.515	1.02
Wood a (54)	1.36	2.42	0.67	1.37	0.132	0.598	1.14
Orange peel (55)	0.76	1.25	0.38	0.75	0.526	0.236	2.26
Wood b (56)	1.36	2.53	0.79	1.63	0.673	0.351	0.90

## References

- [1] M. Oren and S.K. Nayar, "Generalization of the Lambertian Model and Implications for Machine Vision," *Int'l J. Computer Vision*, vol. 14, no. 3, 1995, pp. 227-251.
- [2] K.E. Torrance and E.M. Sparrow, "Theory for Off-Specular Reflection from Roughened Surfaces," *J. Optical Society of America*, vol. 57, no. 9, 1967, pp. 1105-1114.
- [3] X.D. He, K.E. Torrance, F.X. Sillion and D.P. Greenberg, "A Comprehensive Physical Model for Light Reflection," *ACM Computer Graphics*, vol. 25, 1991, pp. 175-186.
- [4] E.P.F. Lafortune, S. Foo, K.E. Torrance and D.P. Greenberg, "Non-linear Approximation of Reflectance Functions," *Proceedings of SIGGRAPH*, 1997, pp. 117-126.
- [5] L.B. Wolff, S.K. Nayar and M. Oren, "Improved Diffuse Reflection Models for Computer Vision," *Int'l J. Computer Vision*, vol. 30, no. 1, 1998, pp. 55-71.
- [6] P. Beckmann and A. Spizzichino, *The Scattering of Electromagnetic Waves from Rough Surfaces*, Pergamon, New York, 1963.

- [7] J. Stam, "Diffraction Shaders," *Computer Graphics, Proceedings of SIGGRAPH 99*, ACM Press, New York, 1999, pp. 101-110.

Table 3: Total least squared errors for the models studied and the surface samples used.

Surface sample (no)	VH-G	VH-E	FC-G	FC-E	ON	LFTG
Felt (1)	23.08	23.05	4.05	3.79	12.52	9.83
Polyester (2)	25.36	25.40	3.89	3.67	11.47	8.69
Leather (5)	15.55	15.49	6.26	5.82	11.88	6.56
Velvet (7)	2.87	3.51	12.57	10.60	29.39	15.46
Frosted glass (9)	2.59	2.04	1.94	1.15	44.78	17.00
Rough paper (12)	24.72	24.67	5.44	5.27	8.92	7.80
Cork (16)	3.92	3.84	3.05	2.14	28.17	14.24
Rough tile (17)	4.19	4.03	4.61	3.45	30.22	8.68
Styrofoam (20)	18.66	18.66	2.82	2.48	13.91	9.74
Lambs wool (22)	9.09	9.12	3.08	2.44	17.40	13.29
Lettuce leaf (23)	2.86	2.53	3.13	2.15	40.31	10.68
Quarry tile (25)	2.00	1.57	2.12	1.35	44.74	14.95
Z polyester (29)	18.89	18.89	3.99	3.65	12.44	8.82
Z rough paper (31)	21.55	21.49	4.42	4.19	10.26	8.55
Slate a (33)	4.87	4.38	1.95	1.40	43.50	18.77
Slate b (34)	0.81	0.57	2.05	1.48	55.58	9.55
Limestone (36)	7.51	7.44	3.84	3.06	20.16	9.67
Brick a (37)	28.28	29.33	4.13	3.95	9.21	12.87
Human skin (39)	5.14	4.98	2.14	1.35	29.20	12.64
Brick b (41)	1.42	1.16	2.15	1.60	55.46	10.86
Linen (44)	7.37	7.16	1.59	0.96	25.57	13.66
Wood a (54)	6.27	6.07	2.56	1.75	32.10	14.73
Orange peel (55)	3.14	2.78	3.32	2.22	36.07	9.96
Wood b (56)	14.17	14.07	4.95	4.44	16.52	11.80

- [8] C.L. Vernold, and J.E. Harvey, "A Modified Beckmann-Kirchoff Scattering Theory for Non-paraxial Angles," *Proceedings of the SPIE*, vol. 3426, 1998, pp. 51-56.
- [9] M. Nieto-Vesperinas and N. Garcia, "A Detailed Study of the Scattering of Scalar Waves from Random Rough Surfaces," *Optica Acta*, vol. 28, no. 12, 1981, pp. 1651-1672.
- [10] J.A. Ogilvy, *Theory of Wave Scattering from Random Rough Surfaces*, Adam Hilger, Bristol, 1991.
- [11] H. W. Jensen, S. Marschner, M. Levoy and P. Hanrahan, "A Practical Model for Subsurface Light Transport," *Proceedings of SIGGRAPH*, 2001, pp. 511-518.
- [12] CURET database, [www.cs.columbia.edu/CAVE/curet](http://www.cs.columbia.edu/CAVE/curet).
- [13] P. Beckmann, "Scattering of Light by Rough Surfaces," *Progress in Optics: ed. E Wolf*, vol. VI, 1967, pp. 55-69.
- [14] H. Ragheb and E.R. Hancock, "Estimating Surface Characteristics using Physical Reflectance Models," *Proc. IEEE Computer Vision and Pattern Recognition*, 2003, pp. 177-184.
- [15] Y.L. Sun, "Statistically-based reflection model for rough surfaces," *Proc. of the SPIE: Computational Imaging*, vol. 5016, 2003, pp. 91-102.
- [16] W. Matusik, H. Pfister, M. Brand and L. McMillan, "A Data-Driven Reflectance Model," *ACM Transactions on Graphics*, vol. 22, no. 3, 2003, pp. 759-769.
- [17] H. Ragheb and E.R. Hancock, "Adding Subsurface Attenuation to the Beckmann-Kirchoff Theory," *Iberian Conf. Pattern Recognition Image Analysis*, 2005, LNCS 3523, pp. 247-254.

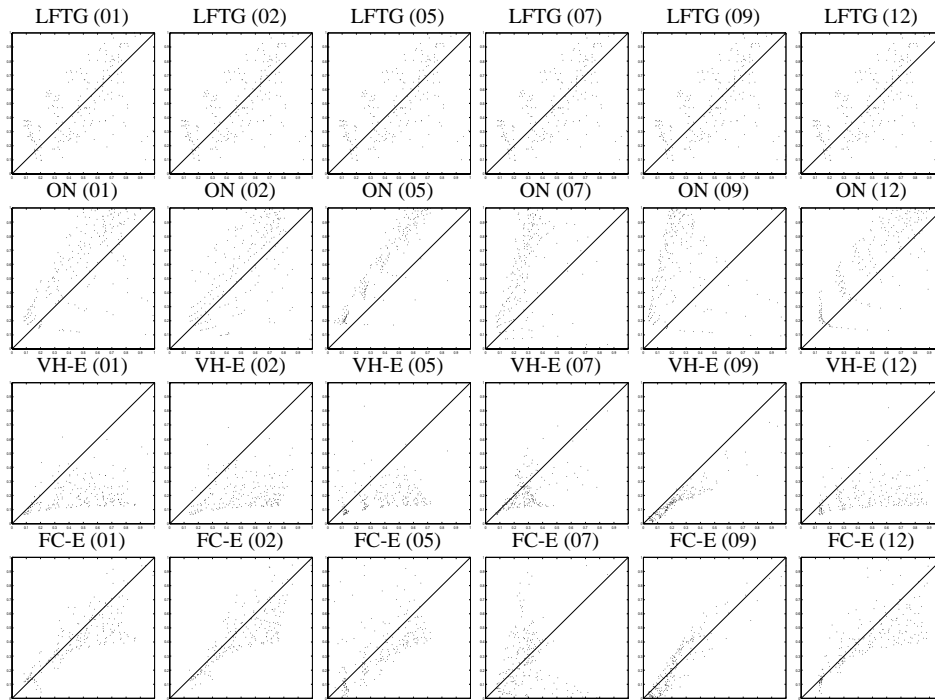


Figure 3: Results for the surface samples 1, 2, 5, 7, 9 and 12: normalized radiance predicted by the model (Table 1) versus normalized radiance data derived from the BRDF measurements.

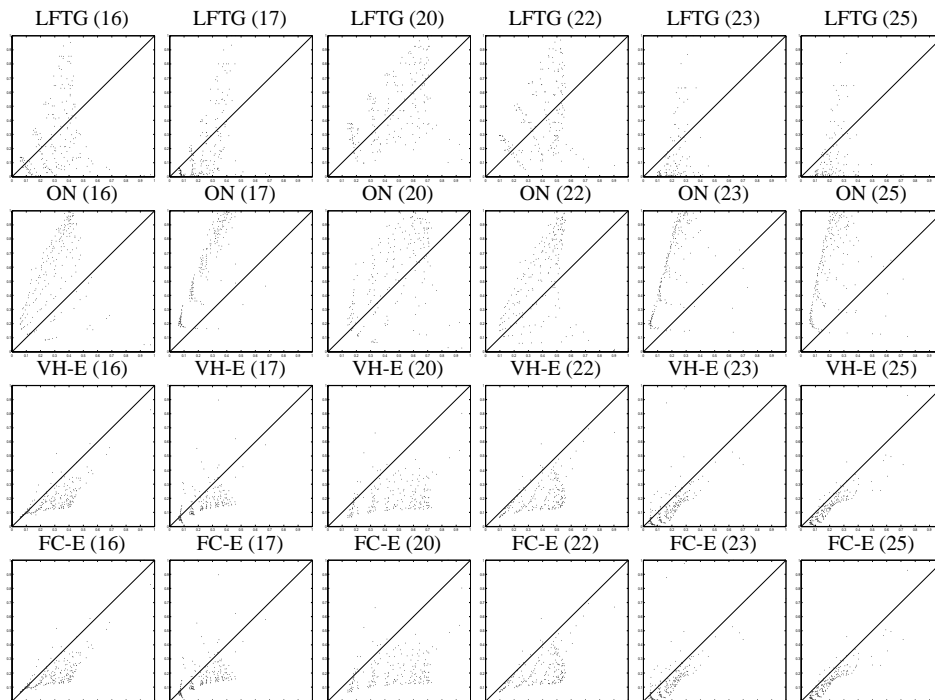


Figure 4: Results for the surface samples 16, 17, 20, 22, 23 and 25 (as described in Fig. 3).

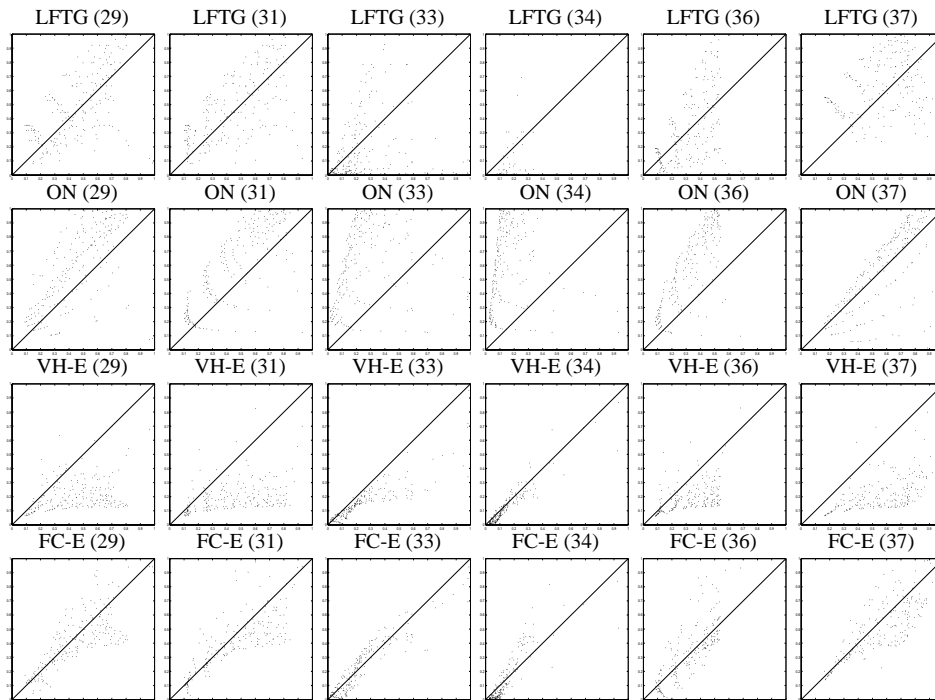


Figure 5: Results for the surface samples 29, 31, 33, 34, 36 and 37 (as described in Fig. 3).

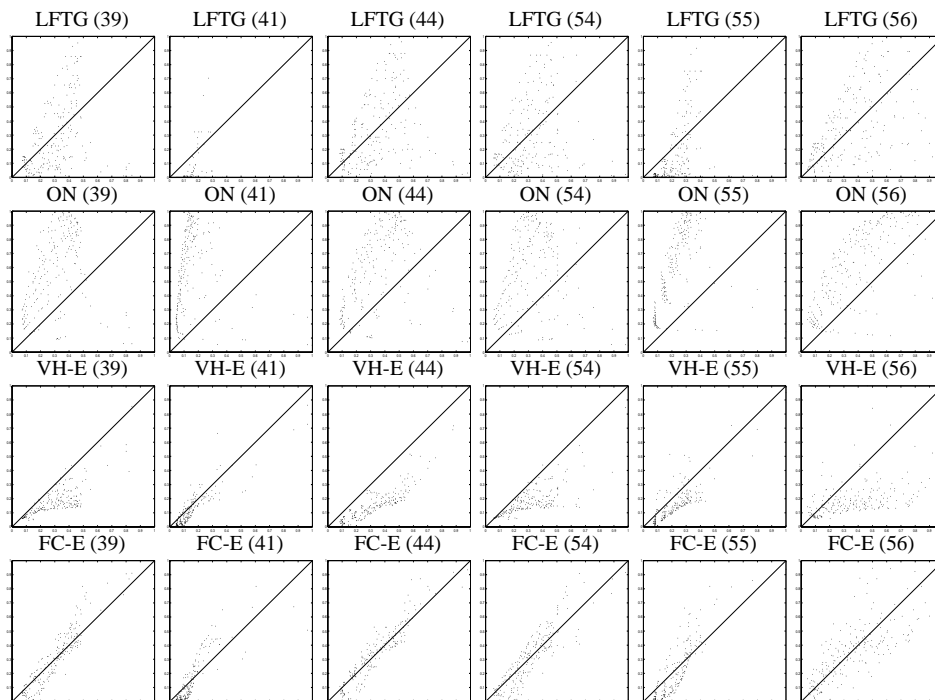


Figure 6: Results for the surface samples 39, 41, 44, 54, 55 and 56 (as described in Fig. 3).