Optical Measurement by Back Projection

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In this paper we first discuss a particular approach suggested for feature inspection, namely 'optical back projection', and then the results of model calculations to determine the robustness of the scheme.

One of the tasks specified for the vision group of the RIPR project is the implementation of a run time inspection cell which is able to identify a small object, and to subsequently inspect it using a single camera, referred to as the inspection camera, attached to a robot arm. As a definite example, we may want to determine whether a hole has been machined on a particular face of the object with the correct radius and at the correct position, at least to within some previously agreed upon tolerance.

As has been implied, measurements of features will assumed to be given with respect to some set of accurately known features (referred to as calibration features) on the object (such as its edges), ie the body of the object is taken to have been manufactured accurately, any imperfections are assumed to exist solely in the finer detail. The inspection cell demonstrator will thus be envisaged to be used for feature verification.

Optical Back Projection

As explained, we wish to determine various properties of some feature which is taken to lie on a given planar surface of the object being inspected.

It is convenient to introduce three important reference frames for our discussion, namely the world, the inspection and the camera frames.

Figure 1 depicts the experimental set up for the inspection task, where the object is assumed to have been placed at some position in the world frame. The feature to be inspected is assumed to lie in the plane Λ, referred to as the feature plane.

We can see from the figure that the feature plane may be determined simply by choosing three suitable non-collinear points P1, P2 and P3. These three points define a plane which the feature lies in, and it is straight forward to determine a right-handed co-ordinate system with P2 as the origin, this defines the inspection frame.

Finally, from the assumed presence of suitable calibration features on the object, it is possible to determine the location of the camera frame with respect to the world frame, and since the inspection frame is well determined, we can therefore determine the location of the inspection camera with respect to the inspection frame.

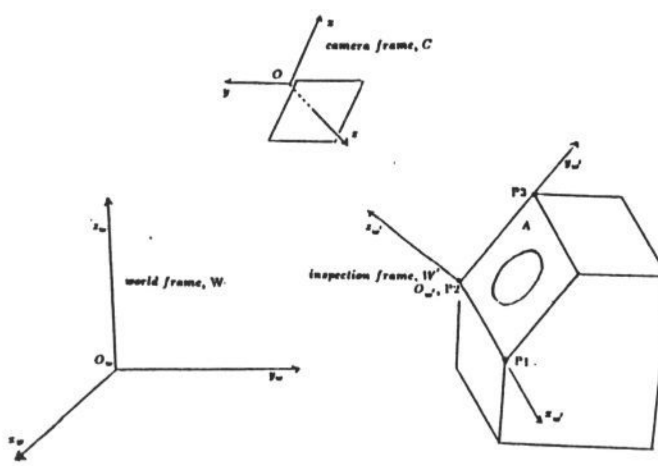

Figure 1: The relevant reference frames for feature inspection.

Before discussing further optical back projection, we must digress somewhat and discuss the relationship between the co-ordinates of points in the camera frame and the co-ordinates in the image plane of the inspection camera. We also discuss the importance of viewpoint consistency.

The camera model

The model for the camera adopted here is equivalent to the pin-hole camera model, but places the point of projection behind the image plane which has the advantage that the image occurs right side up (see Ballard and Brown [1]). Mathematically the transformation involved is called a perspective projection.

The perspective projection gives the image plane co-ordinates of the point (x,y,z) as

$$u = fx/z, \ v = fy/z,$$

where f is referred to as the focal length.

The camera model is developed further by including

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radial distortion, image centre off-set and taking into account the different sampling rates of the camera and the A/D converter (for a full account of these effects see Tsai [2] and Lenz and Tsai [3]). It is important to note that given the measurements U, V it is possible to correct for the effects mentioned above to determine the image co-ordinates u, v which result from the perspective projection.

Viewpoint Consistency

Important to our approach is the necessity of determining the location of the inspection camera in the world frame, this is referred to as 'viewpoint consistency'. We determine the location of the camera in the world by making use of the assumed presence of suitable calibration features possessed by the object and its appearance in the image plane of the inspection camera.

In our work we have implemented two viewpoint consistency schemes, one based upon the work of Tsai [2], and the other due to Lowe [4], [5].

One of the advantages of the Lowe scheme over others is that, besides incorporating point-point correspondence it also makes use of line-line correspondence. The inclusion of line-line correspondences is important, mainly because, as Lowe notes, low-level vision routines are relatively efficient at determining the transverse locations of lines, but are much less certain about where exactly the lines terminate. The scheme does have the dis-advantage of requiring an initial estimate of the current location of the inspection camera, but this is not too serious in our use, since this may be obtained, for example, by interrogating the robot.

We note the viewpoint consistency scheme based upon the work of Tsai is an algebraic, as distinct from an iterative, method.

We now return to our discussion of back projection.

Point projection

Thus, as explained, from the measured image co-ordinates U, V, it is possible to determine the undistorted image co-ordinates u, v, which are related to the co-ordinates (x,y,z) of a point P in the camera frame by the perspective projection as discussed above. We have also shown that it is possible to determine the relative locations of the camera and inspection frames. For the following discussion it is convenient to drop the prime on inspection frame quantities and to refer to the inspection frame as the world frame.

The relationship between measurements in the camera and world frames is taken as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix} + T,$$

where R describes the relative orientation of the reference frames and T their relative position (see Fu et al [6]).

We show in figure 2 the relationship between the camera and world frames. The figure also shows the projection of a point (u,v) on the image plane onto the z_w = 0 plane, ie the feature plane.

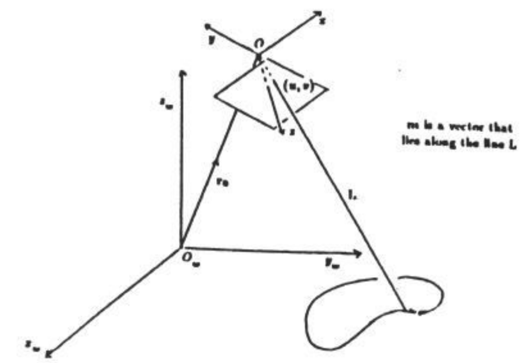

Figure 2: Projecting image points onto the the feature plane

Referring to figure 2, any point on the line L has the form

$$\mathbf{r} = \mathbf{r}_0 + \lambda \mathbf{m},$$

where $\mathbf{r}_0$ is any point on the line, conveniently taken as the point O, and $\mathbf{m}$ is a vector in the direction of the line.

The point O has world co-ordinates determined from

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = R\mathbf{r}_0 + T,$$

thus giving

$$\mathbf{r}_0 = -R^{-1}T,$$

taking some liberty with the notation.

The image point (u,v) has camera co-ordinates $(u,v,f)^t$ and thus world co-ordinates of

$$R^{-1} \begin{pmatrix} u \\ v \\ f \end{pmatrix} - R^{-1}T,$$

and therefore the world co-ordinates of a vector lying in the direction of the line L are given as

$$\mathbf{m} = R^{-1} \begin{pmatrix} u \\ v \\ f \end{pmatrix}.$$

Given that we know the equation of the line L, it is straight forward to determine the point of intersection of this line with the feature plane, Λ.

The plane Λ is determined from

$$\mathbf{r}.\mathbf{n}_w = p_w,$$

where $\mathbf{n}_w$ is a vector perpendicular to the feature plane and p_w the perpendicular distance from the origin to the plane. The feature plane has

$$\mathbf{n}_w = \mathbf{k}, \quad p_w = 0,$$

where $\mathbf{k}$ is a unit vector lying along the world z axis. The intersection of the line L with the plane Λ is determined once λ is given. We have

$$\mathbf{r}_0.\mathbf{n}_w + \lambda \mathbf{m}.\mathbf{n}_w = p_w,$$

ie

$$\lambda = \frac{p_w - \mathbf{r}_0.\mathbf{n}_w}{\mathbf{m}.\mathbf{n}_w},$$

giving the point of intersection as

$$\mathbf{r}_p = \mathbf{r}_0 + \frac{p_w - \mathbf{r}_0.\mathbf{n}_w}{\mathbf{m}.\mathbf{n}_w}\mathbf{m},$$

and $\mathbf{m}.\mathbf{n}_w = 0$ only in the un-interesting case when the inspection camera is looking at the feature edge on.
In this manner, we can project all the image points of the feature from the inspection camera image plane back onto the feature plane.

Program Outline

We now give a general overview of the approach adopted here for feature inspection using optical back projection. Figure 3 attempts to give the salient features of our approach:

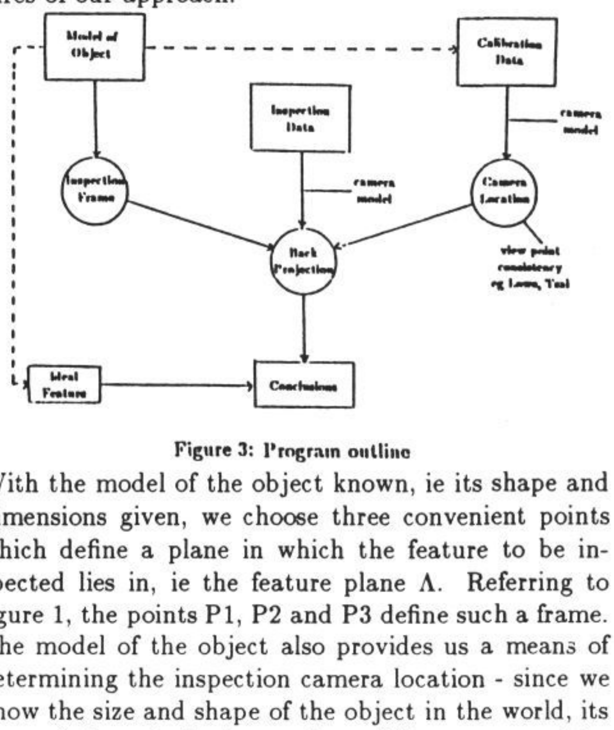

Figure 3: Program outline

With the model of the object known, ie its shape and dimensions given, we choose three convenient points which define a plane in which the feature to be inspected lies in, ie the feature plane Λ. Referring to figure 1, the points P1, P2 and P3 define such a frame. The model of the object also provides us a means of determining the inspection camera location - since we know the size and shape of the object in the world, its size and shape in the image plane of the camera enables us to determine its location - ie we can determine the camera viewpoint.
Given that we know the locations of both camera in the world and the inspection frame in the world, it is an easy matter to determine the location of the camera relative to the inspection frame. We can then correct for the camera properties such as focal length, radial distortion, $\cdots$, etc, and its location (ie the perspective projection), to project the image of the feature back through the camera system and back onto the plane the feature is assumed to lie in, ie the feature plane (see figure 2).
Of course, we can imagine the inspection camera being moved to several positions, and optimally combining our results with, for example, a Kalman filter to deduce a minimum error estimate of the measurements (see Gelb [7]).

We note that this last step is equivalent to keeping the camera fixed and moving the object, ie only relative motion between object and camera is important. We have adopted the particular choice of moving the camera in the world, though there may be occasions when it is more convenient to consider the object to have moved in the world, for example, the object may be placed on a turn-table.
Finally, when we have completed taking measurements, we move to the final step which considers the final values for the inspection feature parameters and compares them with the respective ideal feature parameters. A decision is then made as to whether, to within some previously agreed upon tolerance, the projected feature and ideal feature are the same.

Results and Discussion

We have discussed an implementation of a program to determine the properties of features which lie on a well defined plane of an object using optical back projection. For part of this task it is necessary to determine the location of the inspection camera with respect to the feature being inspected, and we discussed the use of two viewpoint consistency schemes, one based on the camera calibration technique of Tsai and the other due to Lowe.
We now discuss model results for inspecting a circle of radius 5.0 mm which we imagine has been machined on a particular surface of a model object at a given position, see figure 4.

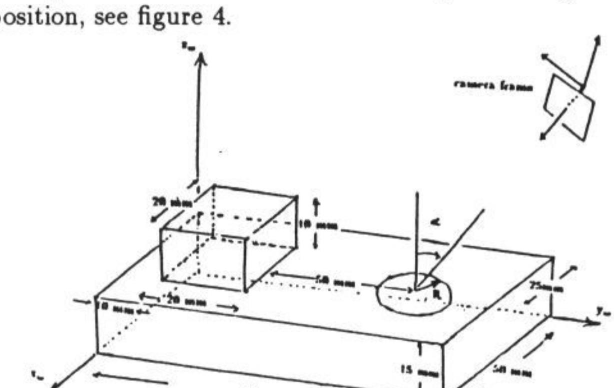

Figure 4: The model object

Thus, given an assumed viewpoint of the inspection camera it is straight forward to generate the calibration image data. The inspection data, ie the image data for the circle feature, is generated in the same manner. By introducing various model distortions acting on perfect data it is possible to determine their effect in the context of the inspection scheme being employed, ie whether the neglect of the effect is reasonable and causes only small acceptable inaccuracies in the inspection process.
For example, a working assumption will be that the focal length of the camera is known. However, if the camera has been calibrated for one lens setting, then if the

lens setting has been changed the focal length assumed will be inappropriate. Now, if the assumed value of the focal length is incorrect, the camera mis-interprets the (calibration) image data and believes that it is at a different location in the world than it really is. It thus follows that the relative position the camera believes the inspection plane is at is also incorrect. But the value of the focal length is also needed to be able to project the inspection data back onto the inspection plane. It is not obvious how this interplay will affect the inspection scheme, since we are really only interested in the relative positions of features in the inspection plane, eg referring to figure 4, we are only interested in the position of the circle relative to the edges of the object.
We report here the results of Mc Tavish and Mayhew [8], who divided their investigations into four broad areas:

intrinsic parameters,
accuracy of calibration data,
inspection plane distortion,
accuracy of image data.

We first note that since Lowe is an iterative method, convergence to the solution is not guaranteed even for perfect data. The model results suggest that, however, the scheme works well with perfect data, the radius and centre of the circle being reproduced to about 0.1 - 0.2 % for viewing angles $\alpha \lesssim 85^o$, then increasing to about 0.3% for the radius determination and about 2.0% for the centre determination for $\alpha \approx 89^o$ (in figure 4, the angle α is referred to as the viewing angle and it conveniently represents the angle the camera optical axis makes with the normal to the inspection plane).
Restricting our discussion to the determination of the radius of the circle, we note that if we wish the circle to be reproduced to within 1 - 2%, the focal length must be known accurately to within about 5%, though if we increased the allowable error in the radius determination, quite large errors in the focal length are admissible.
Both the modelling of radial distortion and the presence of image centre off-set had little effect, generally well within our 1 - 2% limit for quite large values of parameter errors introduced.
There seemed to be little variation in determining the radius with viewing angle for a given value of horizontal scale factor when it is known to within about 5%, except at the larger viewing angles.
Generally, the object should be manufactured as accurately as possible, certainly to within about 0.2 mm, while the lack of planarity of the plane the feature lies in should be kept to less than about 0.1 - 0.5 mm. Image data should be accurate to within about $\frac{1}{10}$ - $\frac{1}{2}$ PIXEL (though we have not included digitisation of the spatial co-ordinates in the camera model, the parameter PIXEL is introduced to be of 'typical' pixel size, we take PIXEL = 0.010 mm).
We also note that, except at the largest viewing angles considered, both Lowe and Tsai worked quite well, certainly comparable with each other. Lowe seemed to fare particularly badly at the larger viewing angles, but this is probably not too serious, since we may expect to use our inspection scheme under the most appropriate viewing conditions.

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